The Fundamental Surplus Strikes Again

Lars Ljungqvist Thomas J. Sargent*

April 15, 2021

Abstract

The fundamental surplus isolates parameters that determine how sensitively unemployment respond to productivity shocks in the matching models of Christiano, Eichenbaum, and Trabandt (2016 and this issue) under either Nash bargaining or alternating-offer bargaining. Those models thus join a collection of models in which diverse forces are intermediated through the fundamental surplus.

KEY WORDS: Matching model, alternating offer bargaining, fundamental surplus, DSGE, unemployment, business cycle.

*Ljungqvist: Stockholm School of Economics and New York University (email: lars.ljungqvist@hhs.se); Sargent: New York University and Hoover Institution (email: thomas.sargent@nyu.edu). We are grateful for excellent research assistance by Quentin Batista and Zejin Shi. Larry Christiano, Marty Eichenbaum and Mathias Trabandt generously shared their computer codes with us. We thank an anonymous referee for helpful suggestions. Ljungqvist's research was supported by a grant from the Jan Wallander and Tom Hedelius Foundation.

1 Introduction

To generate big responses of unemployment to productivity changes, matching models have been reconfigured in a variety of ways. Thus, Shimer (2005, p. 26) showed that to explain observed movements in unemployment and vacancies, "one must assume that the value of leisure is nearly equal to labor productivity," a line of inquiry pursued with calibrations by Hagedorn and Manovskii (2008). Pissarides (2009, p. 1341) advocated departing from the standard assumption that matching costs are proportional to the duration of a vacancy: "a simple remodeling of the costs from proportional to partly fixed and partly proportional can increase the volatility of [labor market variables], virtually matching the observed magnitudes." Wasmer and Weil (2004, p. 944) formulated a two-layer matching model in which firms must first match with bankers to obtain credit before matching with workers, and demonstrated "that credit frictions amplify [unemployment] volatility through a financial accelerator." Hall and Milgrom (2008, p. 1673) abandoned Nash bargaining in favor of alternating-offer bargaining (AOB) that converts threat points from outside values into values from continued bargaining; that led them to conclude that "the limited influence of unemployment on the wage results in large fluctuations in unemployment under plausible movements in [productivity]." Given their diversity, one might be tempted to think that it would be fruitless to seek a single common channel through which the forces unleashed in all such reconfigurations must operate.

Remarkably, transcending these and many other matching models, there is a single intermediate channel – the fundamental surplus – through which economic forces affecting the responsiveness of unemployment to changes in productivity are funneled. The *fundamental surplus* is the difference between productivity y and a model-specific quantity x, while $(y-x)/y$ is called the fundamental surplus *fraction*. The fundamental surplus determines one factor in a two-factor multiplicative decomposition of the elasticity of market tightness with respect to productivity.[1] The decomposition comes from a comparative steady state analysis. The first factor satisfies a quantitatively small upper bound coming from a professional consensus about values of the exogenous elasticity of matching with respect to unemployment. That means that the second factor – the inverse of the fundamental surplus fraction – is the principle determinant of the elasticity of market tightness and hence, of how sensitively unemployment respond to productivity. A decomposition like this was first con-

[1] In matching models, market tightness is the endogenous ratio of vacancies to unemployment, movements in which affect the number of matches formed and the level of unemployment.

structed by Shimer (2005) and Hagedorn and Manovskii (2008) in the context of a standard matching model with Nash bargaining and basic features. Ljungqvist and Sargent (2017) discovered and named the fundamental surplus by deriving such two-factor multiplicative decompositions for a wide class of matching models.

That the fundamental surplus shows up as an "uninvited guest" in a variety of models indicates its role in unveiling essential economic forces driving outcomes in an interesting *class* of models. It points like a laser at the heart of matching models: resources assigned to create vacancies. The fundamental surplus fraction is an upper bound on the fraction of a job's output that the *invisible hand* of market forces can allocate to vacancy creation. Thus, take a standard matching model with Nash bargaining. Here x is the value of leisure, as originally computed by Shimer (2005) and Hagedorn and Manovskii (2008). To motivate workers, the invisible hand has to award them at least the value of leisure; so that value cannot be allocated to vacancy creation. When Pissarides (2009) adds a fixed matching cost, it brings another deduction that must be made to arrive at the fundamental surplus. This deduction is based on an annuity payment that has the same expected present value as the fixed matching cost. Hence, in addition to the required compensation for lost leisure, the invisible hand's allocation of resources to vacancy creation is now constrained also to accommodate the fixed matching cost.[2] Similar reasoning leads to the conclusion that an extra deduction to arrive at the fundamental surplus in the financial accelerator model of Wasmer and Weil (2004) entails the annuitised value of average credit search costs that firms incur before matching with workers. In the AOB model of Hall and Milgrom (2008), a novel finding is that a firm's cost of delay in bargaining suppresses the fundamental surplus. That might seem odd at first since no such cost is incurred in an equilibrium because the parties immediately reach an agreement. However, that agreement reflects how workers strategically exploit the firm's cost of delay under the alternating-offer bargaining protocol. Hence, the invisible hand's allocation of resources to vacancy creation is bounded both by the required compensation for lost leisure and by what workers attain under the AOB

[2]When the fixed matching cost is incurred *after* bargaining, the associated deduction in the fundamental surplus is the described annuity payment; but if the firm incurs the fixed matching cost *before* bargaining as in Pissarides (2009), that deduction is amplified by a factor equal to the inverse of the firm's Nash bargaining weight. (See Ljungqvist and Sargent (2017, sec. III.C) and accompanying online appendix A.5, respectively.) In the first case, because the cost simply reduces the match surplus, the fixed matching cost is born jointly by the firm and the worker. In the second case, the fixed matching cost must *ex ante* be financed out of the firm's share of the match surplus. Hence, the smaller is the firm's share, the more the match surplus must increase in order for the firm to cover the cost; that enlarges the associated deduction in the fundamental surplus.

protocol as functions of a firm's cost of delay in bargaining. Derivations and interpretations of fundamental surpluses in more matching models appear in Ljungqvist and Sargent (2017) and an accompanying online appendix.[3]

In all but one of the matching models analyzed by Ljungqvist and Sargent (2017), only the first factor in the multiplicative decomposition contains endogenous variables, i.e., the factor mentioned above that satisfies a quantitatively small upper bound. Hence, parameters that shape the second factor – the inverse of the fundamental surplus fraction – are the critical determinants of the elasticity of market tightness. The exception is the Hall-Milgrom AOB model. However, under the assumption that the exogenous job destruction probabilities under bargaining and production are the same, the Hall-Milgrom AOB model also features a second factor without endogenous variables. Imposing this parameter restriction, we produce what we call the *approximating version* of the Hall-Milgrom AOB model. This version attains the ultimate Hall-Milgrom outcome not only of a limited but of a complete lack of "influence of unemployment on the wage" because a worker's outside value then vanishes. The approximating version of the Hall-Milgrom AOB model isolates the value of leisure and a firm's cost of delay in bargaining as the critical parameters for determining how sensitively unemployment respond to productivity. This is confirmed in stochastic simulations of the model across the unconstrained parameter space (i.e., without the simplifying parameter restriction). Actually, any parametric differences in job destruction probabilities during bargaining versus production acts like nuisances when seeking to identify the truly critical parameters determining the elasticity of market tightness. Yes, a difference in job destruction probabilities does affect the elasticity but only if the critical parameters are such that the elasticity would be high in any case; but on its own, the parameterization of a difference in job destruction probabilities cannot yield a high elasticity without the support of the two critical parameters, i.e., the value of leisure and a firm's cost of delay in bargaining. In contrast, either the value of leisure or a firm's cost of delay in bargaining can be calibrated high enough to generate any market tightness elasticity regardless of the calibration of other

[3]To derive a fundamental surplus in a new matching model, one has to settle on components to be deducted from productivity. To find correct recipes for these deductions as well as to interpret them in terms of economic forces can occasionally be challenging; but ultimately, an enlightening derivation will prove helpful in parameterizing, estimating, and simulating a model. So far, all deductions that we have derived in diverse matching models have lent themselves to being interpreted as set-asides that the invisible hand cannot allocate to vacancy creation. The Nash bargaining model and the AOB model of Christiano, Eichenbaum, and Trabandt (2016 and this issue) are no exceptions. For example, why do the deductions associated with the fixed matching cost involve the annuitised value of that cost in their Nash bargaining model but the full value in their AOB model? Section 4 of this paper provides the answer.

parameters.

In this paper, we show that the matching models of Christiano, Eichenbaum, and Trabandt (2016 and this issue), henceforth CET, join the class of models for which the fundamental surplus is the key intermediating quantity. The analysis of CET's Nash bargaining model is almost identical to that of the Pissarides (2009) model, and CET's AOB model can be analyzed in the same way as can the Hall-Milgrom AOB model, only now with the addition of a fixed matching cost. It is remarkable how well the fundamental surplus isolates critical primitives when matching in the labor market is incorporated into CET's DSGE model with habit formation in preferences, adjustment costs in capital formation, capacity utilization costs, Calvo sticky price frictions, and a Taylor rule for monetary policy. Thus, for the AOB model, to attain what CET would view to be an empirically plausible model, it is necessary to assemble a combination of high values of the value of leisure, a firm's cost of delay in bargaining, and the fixed matching cost. If those parameters are set too low, the fundamental surplus fraction won't be small enough to make unemployment respond sensitively to productivity. Furthermore, to illustrate how different job separation probabilities during bargaining and production are just distractions in this context, we set them equal to each other and recalibrate a critical parameter of the fundamental surplus in order to generate the same fit to data as CET's estimated model. Thus, this alternative parameterization is an example of a configuration that we above called an *approximating version* of the model. It generates virtually identical impulse-response functions with respect to 12 variables and three types of shocks as those computed by CET for the estimated model.[4]

[4]We presented our findings here to Larry Christiano, Marty Eichenbaum, and Mathias Trabandt shortly after the appearance of their earlier working paper (Christiano et al. 2020). Fortunately, from their working paper, they have deleted an incorrect claim that the fundamental surplus somehow challenges the possibility that wage inertia is important. To the contrary, in a model of wage inertia like the AOB model, wage inertia can be a powerful mechanism for making unemployment respond sensitively to productivity, provided that the fundamental surplus fraction is calibrated to be small enough. Despite that improvement relative to their working paper, CET (this issue) continue to disagree with us about the usefulness of the fundamental surplus and how it applies to their work. Thus, consider these two questions: (1) Are the value of leisure, a firm's cost of delay in bargaining, and the fixed matching cost the critical parameters that determine how sensitively unemployment respond to productivity in CET's AOB model? And (2) is the difference in the probabilities of job destruction during bargaining versus production just a side-show in terms of the task of identifying those critical parameters? Our present paper explains how the fundamental surplus tells us that the answers to both questions are "yes". Beyond that, we learn little from CET's wish list and taxonomy of decompositions of the elasticity of market tightness because they do not help isolate critical parameters that determine how sensitively unemployment respond to productivity. We also find unenlightening CET's attempt to criticize the fundamental surplus by exogenously imposing an *ad hoc* law of motion for the wage rate. That seems to throw the baby out with the bath water: how endogenous wage outcomes are influenced by deep parameters characterizing preferences and opportunities are central to what makes matching models tick.

For readers who want to see a model in which the fundamental surplus is *not* central, Kehoe, Lopez, Midrigan, and Pastorino (2019) present a matching model in which they correctly claim to "have abstracted from the standard mechanism of *differential productivity across sectors* [market production versus home production and vacancy creation]." A fruitful way to get under the hood of the Kehoe et al. model is to interpret their assumptions about parameters in terms of our decomposition of the elasticity of market tightness. First, they assume that key quantities are proportional to productivity, in particular, quantities that are deducted from productivity when calculating the fundamental surplus; this causes our second multiplicative factor comprising the fundamental surplus to vanish. And since the first multiplicative factor is bounded from above, it follows that the elasticity cannot be large (a point anticipated by Ljungqvist and Sargent (2017, p. 2664, footnote 28)). Second, assuming that a firm's cost of posting a vacancy is proportional to productivity makes the elasticity become zero. As a result of these assumptions, under standard preferences with constant relative risk aversion Kehoe et al. find that unemployment is unresponsive to productivity. To alter that outcome, they assume preferences that make risk aversion vary over a business cycle; their baseline model uses a version of Campbell-Cochrane (1999) preferences. Thus, while Kehoe et al. contribute to a literature on how assuming what David Backus mischievously called "exotic preferences" can generate fluctuations,[5] their analysis is not within the general class of matching models with "standard preferences" maintained by CET.

A prelude in Section 2 accounts for how paths of the CET models and the fundamental surplus first crossed. Section 3 puts all the cards on the table and shows that the fundamental surplus works like a charm for CET. Sections 4 and 5 drill down into details of a proper fundamental surplus analysis of CET's AOB model. An example of how to disarm the fundamental surplus channel is presented in Section 6. Section 7 offers concluding remarks.

[5]Although Backus, Routledge, and Zin (2004) did not list Campbell-Cochrane preferences in their catalogue of "Exotic Preferences for Macroeconomists," in conversations with us, David Backus did include them. Ljungqvist and Uhlig (2015) describe peculiar habit dynamics and welfare implications brought by Campbell-Cochrane preferences. Kehoe et al. acquired those peculiarities when they adopted Campbell-Cochrane preferences with external habit, but they avoided them under their baseline setup in which they assumed that the Campbell-Cochrane habit stock is a function of aggregate productivity instead of consumption. Technically, that makes the shock process do double duty both as productivity shock and as preference shock (intermediated through a Campbell-Cochrane look-alike law of motion). Backus et al. did call "exotic" the Epstein-Zin preferences that Kehoe et al. use to show how with these preferences let their results prevail in setups with: (1) a slow-moving predictable component of productivity as in Bansal and Yaron (2004) and discount factor shocks as in Albuquerque et al. (2016) and Schorfheide et al. (2018); and (2) a slow-moving probability of rare disasters as in Wachter (2013).

2 Historical background

2.1 More complicated environments

To conclude (Ljungqvist and Sargent 2017), we studied more complicated environments that illustrate how the fundamental surplus works in the context of two classes of applications: (1) consequences of alternative welfare state arrangements; and (2) business cycle dynamics. To learn about how the fundamental surplus would work in a good-fitting, state-of-the-art, full-of-bells-and-whistles business cycle model, we chose to study CET's (2016) incorporation of matching in the labor market of their DSGE framework. CET compared empirical findings under two bargaining protocols, Nash bargaining and AOB. Their data analysis teaches us that models under these distinct bargaining protocols work about equally well in the sense that the estimates of the two models generate virtually identical impulse-response functions with respect to 12 variables and three shocks – a neutral technology shock, an investment-specific technology shock and a monetary policy shock. The functions are close to the VAR-based empirical estimates of those same impulse responses. As an example of the virtually identical outcomes in the two estimated models, the solid lines in Figure 1 refer to the impulse responses of unemployment to a neutral technology shock under the alternative bargaining protocols.

But CET note that to get those similar impulse response functions under the two distinct bargaining protocols, they have to adjust a key structural parameter – the value of leisure. For their Nash bargaining model, CET estimate a value of leisure that is more than twice what it is in their AOB model. From the perspective of their prior (based on unemployment insurance replacement rates), CET argued that the higher value of leisure estimated under their Nash bargaining protocol is implausibly high. So they proceeded to analyze a *restricted* Nash bargaining model in which they cut the value of leisure roughly in half in order to equate it to what they had estimated for their AOB model. But doing that has adverse consequences: after changing the value of leisure in this way, CET's restricted Nash bargaining model can no longer explain the data. The lower dashed line in Figure 1 shows how unemployment becomes much less responsive to a neutral technology shock. This difference in performance between the estimated and the restricted Nash bargaining model shows that the value of leisure is here a critical parameter. As CET (2016, pp. 1551-1552) correctly inferred, the finding "is reminiscent of Hagedorn and Manovskii's (2008) argument that a high replacement ratio has the potential to boost the volatility of unemployment [... in ...] matching models with Nash bargaining." But CET (2016, p. 1547) misconstrued consequences of their low estimate of

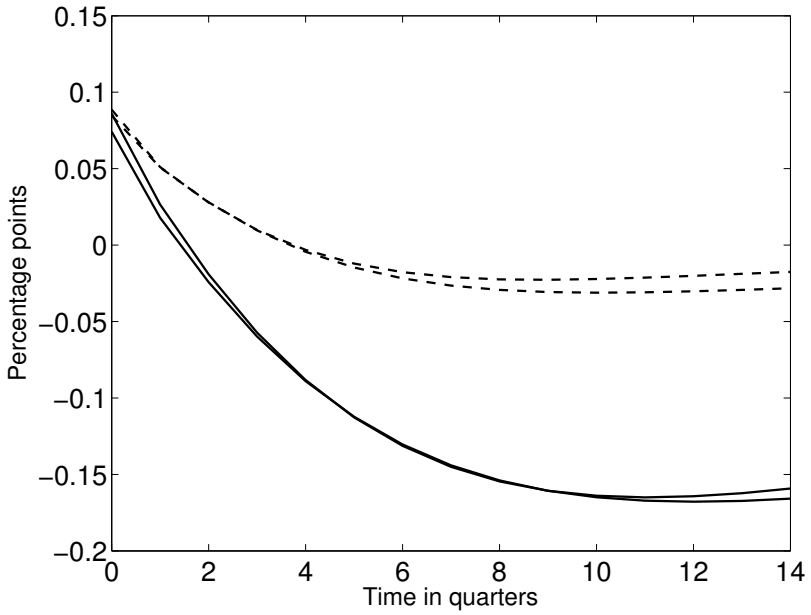

Figure 1: Impulse responses of unemployment to a neutral technology shock in CET's DSGE framework. The solid lines refer to the estimated models with a Nash bargaining protocol and an AOB protocol, respectively. The dashed line refers to the restricted models where parameter values are cut in half for the value of leisure, as well as for a firm's cost of delay in bargaining in the AOB model.

the value of leisure in the AOB model when they said that it meant that "the replacement ratio does not play a critical role in the AOB model's ability to account for the data."

To the contrary, the value of leisure also plays a *critical* role in the AOB model. To understand this, our Section 1 presentation of the fundamental surplus at work in Hall and Milgrom's (2008) AOB model sets the stage. By cutting in half both the value of leisure and a firm's cost of delay in bargaining, we perturbed CET's AOB model to construct what we regard as a counterpart to CET's restricted Nash bargaining model.[6] As reported in Ljungqvist and Sargent (2017, sec. VII.B and online appendix D.2), the *restricted* AOB model also brings a dramatic deterioration in performance, one as bad as that of the restricted Nash bargaining model; hence, the dashed lines depicting a dampened impulse response of unemployment to a neutral technology shock in Figure 1 are almost the same across the two restricted models.

As can be anticipated by the similarities of the solid and dashed lines, in this paper we

[6]Larry Christiano, Marty Eichenbaum, and Mathias Trabandt generously performed this experiment.

show that there is a common intermediate channel – the fundamental surplus – through which the forces affecting the responsiveness of unemployment to productivity operate in both the Nash bargaining model and the AOB model of CET.

2.2 Pertinence of alternative decompositions

There of course exist alternative decompositions of the market tightness elasticity, some more enlightening than others.[7] As discussed in Section 1, the fundamental surplus analysis rests on a two-factor multiplicative decomposition in which an essential feature is that the first factor satisfies a quantitatively small upper bound. Although sometimes this is not possible, an ideal situation occurs when the second factor – the inverse of the fundamental surplus fraction – is expressed solely in terms of parameters. One of CET's (this issue) two alternative decompositions for their Nash bargaining model constitutes such an ideal decomposition, namely, the one that CET call "structural." CET (2020, p. 12) in an earlier working paper characterize that decomposition: "a distinguishing feature of the [fundamental surplus] is that it does not involve endogenous variables," and the "decomposition effectively coincides with the decomposition reported in section A.5 of the [Ljungqvist-Sargent (2017)] online technical appendix," namely, the decomposition for the Pissarides (2009) Nash bargaining model with a fixed matching cost discussed in Section 1.[8] Hence, this is the decomposition that both CET (2020) and we use to analyse CET's Nash bargaining model.[9]

For reasons similar to those in the Hall-Milgrom (2008) AOB model discussed in Section 1, endogenous variables do show up in the second factor of CET's (this issue) two alternative decompositions for their AOB model. Hence, neither of those decompositions are ideal in the sense described above. Nevertheless, one of their decompositions exhibits the essential feature that the first factor satisfies a quantitatively small upper bound, namely,

[7]For an instructive example of alternative decompositions, see Ljungqvist and Sargent (2017, p. 2651, footnote 15) regarding the matching model of Wasmer and Weil (2004). While Petrosky-Nadeau and Wasmer (2013) derive a decomposition that assigns a multiplicative role to the financial accelerator, we show that eliminating endogenous quantities in favor of exogenous ones reveals how the fundamental surplus fraction is the essential determinant. Hence, rather than a multiplicative role, the financial accelerator manifests as an extra deduction that must be made to arrive at the fundamental surplus, as discussed in Section 1.

[8]Specifically, CET's (this issue) "structural" decomposition for their Nash bargaining model has an identical second factor – the inverse of the fundamental surplus fraction – to that of the decomposition for the Pissarides (2009) model derived by Ljungqvist and Sargent (2017, online appendix A.5). However, the two models are not the same – CET assume that a worker can go from one job to another without passing through unemployment – so the first factors differ slightly. But both first factors share the same small upper bound.

[9]Throughout this paper, we adopt our notation from Ljungqvist and Sargent (2017), so our quantities A, α, y, z, c, ϕ, $\gamma/M = \gamma_{\triangle}$ and $1-s$ correspond to CET's σ_m, σ, ϑ, D, s, η, γ and ρ.

the decomposition that CET call "non-structural" and that they first derived in CET (2016, online appendix). In this paper we show that decomposition to be enlightening in terms of its capacity both to identify critical parameters that determine the sensitivity of unemployment to productivity and to explain striking parametric interdependencies in simulations and restricted estimations of CET's AOB model. By way of contrast, ample evidence of how unenlightening the alternative decomposition is can be found in CET (2020) who only use that decomposition in their computations regarding CET's AOB model.

3 Fundamental surplus explains CET

Our fundamental surplus analysis rests on a two-factor multiplicative decomposition of the elasticity of market tightness with respect to productivity. Because the first factor is bounded from above by what is widely agreed to be a small number, it is the second factor – the inverse of the fundamental surplus fraction – that determines how sensitively unemployment responds to productivity. Thus, there is a natural first test of the above decompositions for CET's Nash bargaining model and AOB model, respectively. Specifically, the demonstrated ability of both *estimated* models to explain observed unemployment volatility should manifest in a small fundamental surplus fraction, i.e., the second factor of a decomposition should be large. In contrast, in both *restricted* models that second factor should be much diminished to reflect the inability of these model parameterizations to generate unemployment volatility. Also, across the estimated and restricted version of each matching model, there should not be much of a change in the first factor: it should remain small regardless of a model parameterization's ability or inability to generate unemployment volatility. So let us examine the outcomes.

The first line of Table 1 reproduces the decomposition for CET's estimated Nash bargaining model and the second line presents the decomposition for the restricted model, described in the last two lines of CET's (2020) Table 2. The corresponding decompositions for CET's AOB model are computed on the last two lines of Table 1.[10] As predicted by the fundamental surplus, the second factor – the inverse of the fundamental surplus fraction – explains the changes in the elasticity of market tightness, while the first factor (denoted Υ in Table 1) is small and invariant. The fundamental surplus channel works as follows. Parameters critical for how sensitively unemployment respond to productivity appear as deductions in

[10]The numbers for the estimated AOB model are those of the second line of CET's (this issue) Table 2, but neither CET (2020) nor CET (this issue) analyze a restricted version of the AOB model.

computations of a fundamental surplus. Since a restricted model in Table 1 is derived from the estimated model by cutting some critical parameters in half – the value of leisure in the Nash bargaining model and, in addition, a firm's cost of delay in bargaining in the AOB model, the fundamental surplus is enlarged in the restricted model and hence, the inverse of the fundamental surplus fraction is diminished. That channel for dissipating the model's ability to generate unemployment volatility is shared by the two models in Table 1. Thus, CET's Nash bargaining model and AOB model both provide outstanding examples of the fundamental surplus at work yet again.

Table 1: Elasticity of market tightness with respect to productivity

	Elasticity = $\eta_{\theta,y}$	Υ $\times$	Inverse of the fundamental surplus fraction
Estimated Nash bargaining	20.4	2.11	9.65
Restricted Nash bargaining	3.97	2.11	1.88
Estimated AOB	24.2	1.65	14.66
Restricted AOB	3.00	1.65	1.82

To shed more light on how the fundamental surplus shapes unemployment volatility in CET's stochastic AOB model, we compute a CET version of a figure that Ljungqvist and Sargent (2017, p. 2660, Figure 6) used to analyze the original AOB model by Hall and Milgrom (2008). Starting from CET's estimated AOB model, we perturb a firm's cost of delay in bargaining $\gamma_\triangle$ and the probability δ that a job opportunity is exogenously destroyed between bargaining rounds. Figure 2 shows how different constellations of $(\gamma_\triangle, \delta)$ affect the standard deviation of unemployment. For each pair $(\gamma_\triangle, \delta)$, we adjust a firm's cost of posting a vacancy to make the average unemployment rate stay at 5.5 percent.[11] The circle labelled 'CET' shows the location of CET's estimated AOB model with parameters $\gamma_\triangle = 0.009$ and $\delta = 0.002$. The circle labelled 'restricted' indicates the restricted AOB model except that

[11] Ljungqvist and Sargent (2017) instead adjusted the multiplicative efficiency parameter of the matching function to maintain an average unemployment rate of 5.5 percent. Choosing to use a firm's cost of posting a vacancy and/or the multiplicative efficiency parameter of the matching function to attain the unemployment target affects only vacancy outcomes. As explained in detail by Ljungqvist and Sargent (2017, online appendix C, footnote 33), without targets for vacancy statistics, the choice is immaterial.

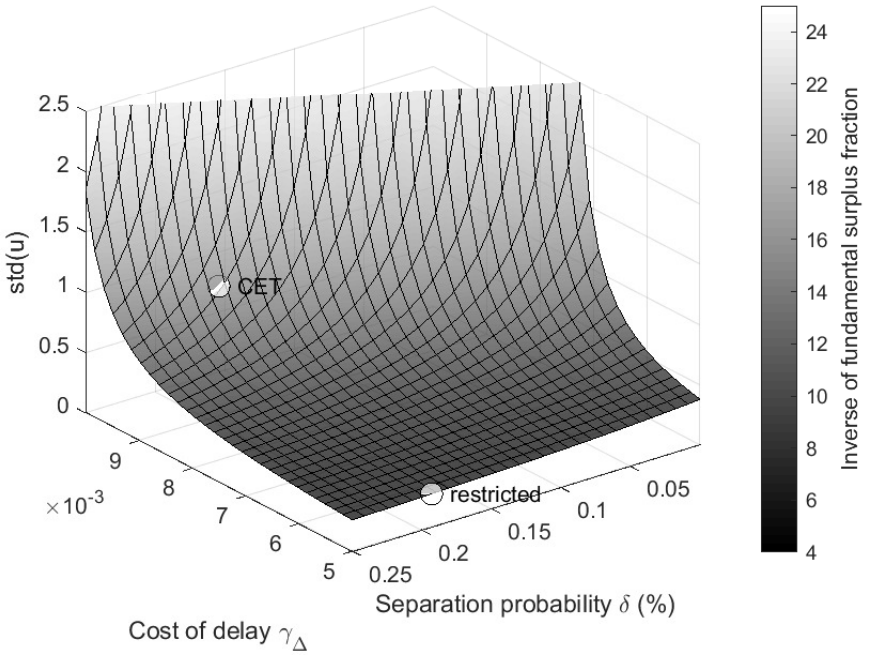

Figure 2: Standard deviation of unemployment in percentage points for different constellations of a firm's cost of delay in bargaining ($\gamma_\triangle$) and the exogenous separation rate while bargaining (δ) in CET's AOB model.

we have reduced $\gamma_\triangle$ only to 0.005 and have left the value of unemployment compensation unchanged.

As indicated by the display to the right of Figure 2, the shade shows the magnitude of the inverse of the fundamental surplus fraction derived by CET (2016, online appendix) and used in our Table 1. Evidently, CET's AOB model cannot generate high unemployment volatility without a small fundamental surplus fraction. Furthermore, Figure 2 conveys that a firm's cost of delay in bargaining is a critical parameter for determining the size of the fundamental surplus fraction. In particular, if the constellation of $(\gamma_\triangle, \delta)$ places a model well within the lowlands in Figure 2, attaining a higher standard deviation of unemployment would require setting a higher value of $\gamma_\triangle$. Our next two sections say more about this graph and what it reveals about the fundamental surplus channel in the AOB framework.

Papetti (2019) shows that the fundamental surplus explains a "ridge" of structural parameters in CET's AOB model that can fit observed unemployment volatility well. He does this by estimating CET's AOB model conditional on various values of a firm's cost of delay

in bargaining. He confirms the presence of an active fundamental surplus force by first uncovering a tight negative linear relationship between the firm's cost of delay in bargaining that he imposes and the value of unemployment compensation that he estimates. Then he computes the fundamental surplus fraction and, as we would expect, finds that it is small and virtually constant across all of his restricted re-estimations of the AOB model. Recall that our multiplicative decomposition of market tightness indicates that the second factor – the inverse of the fundamental surplus fraction – has to be large in order to generate high unemployment volatility. Since Papetti's diverse restricted estimations of the AOB model all require similar high elasticities of market tightness in order to fit observed unemployment volatility, it follows that their associated fundamental surplus fractions must all be small and of similar magnitudes.

The next two sections elaborate on why AOB models pose a special challenge for the fundamental surplus analysis. While some 'art' is required to create appropriate decompositions of the elasticity of market tightness, doing so yields significant insights, as our application to CET's AOB model shows.

4 An approximating version of CET's AOB model

Hall and Milgrom (2008) were the first to put alternating offer wage bargaining in matching models as a way to enhance business cycle dynamics. Rather than Nash bargaining, a firm and a worker take turns making wage offers; during each bargaining round, the threat is to continue to bargain because doing that has a strictly higher payoff than accepting the outside option. After each unsuccessful bargaining round, the firm incurs a cost of delay $\gamma > 0$ while the worker enjoys the value of leisure z. There is also a probability δ that between bargaining rounds the job opportunity is exogenously destroyed and the worker is sent to the unemployment pool.

It is an analytical advantage that the second factor in the two-factor decomposition of the elasticity of market tightness, the primary determinant of the magnitude of the elasticity, is cast solely in terms of parameters. But for AOB, as discussed in Section 1, this is true only when the exogenous job destruction probabilities under bargaining and production are assumed to be equal. Proceeding under that simplifying assumption, Ljungqvist and Sargent (2017, sec. IV.C) analyzed what can be called an "approximating version" of Hall and Milgrom's AOB model in order to arrive at the desired two-factor decomposition of the elasticity. We now do the same for CET's AOB model, including their assumption of a fixed

matching cost κ paid by the firm before bargaining with a newly-found worker.

Instead of Hall and Milgrom's assumption that alternating offers are made in successive periods and can continue indefinitely, CET assume that bargaining proceeds within a period and that there is a maximum number of rounds. Specifically, the firm and the worker take turns extending offers across an even number M sub-periods. The firm goes first and makes a wage offer w_1^f in the first sub-period and, so long as there is no agreement, continues to make offers w_j^f in subsequent odd sub-periods $j = 3, 5, \ldots, M-1$. Likewise, the worker makes wage offers w_j^w in even sub-periods $j = 2, 4, \ldots, M$. An offer w_M^w in the last sub-period is assumed to be take-it-or-leave-it, i.e., the match is broken up if the offer is not accepted.[12]

A sub-period wage rate refers to compensation for work performed in one sub-period. For example, if the first wage offer w_1^f were to be accepted, as it will be in an equilibrium, the match produces output throughout the entire period and the worker's total compensation would be $Mw_1^f \equiv w$. Analogously, on a sub-period basis, a worker's output, the value of leisure and a firm's cost of delay in bargaining, are denoted $y_\triangle \equiv y/M$, $z_\triangle \equiv z/M$, and $\gamma_\triangle \equiv \gamma/M$, respectively. We will now show that the determinant of the fundamental surplus fraction and hence, of the elasticity of market tightness with respect to productivity, is a linear combination of the value of leisure z, a firm's cost of delay in bargaining γ, and the fixed cost κ paid by a firm before bargaining with a newly-found worker.

Since it is optimal for both bargaining parties to make barely acceptable wage offers, the following indifference conditions must hold across two successive equilibrium wage offers.[13] In an odd sub-period j, the equilibrium wage offer w_j^f extended by the firm would make the worker indifferent between accepting the offer or waiting until the next sub-period to make the equilibrium counteroffer w_{j+1}^w:

$$(M-j+1)w_j^f = z_\triangle + (M-j)w_{j+1}^w, \qquad \text{for } j = 1, 3, \ldots, M-1, \tag{1}$$

where the left side is the worker's labor income for the remaining number of sub-periods $M-j+1$ when accepting and working at the wage rate w_j^f, while the right side is the worker's value upon declining the offer which consists of the value of leisure $z_\triangle$ in the current sub-period and subsequent labor income when working for one less sub-period $M-j$ at the counteroffer wage rate w_{j+1}^w (that the firm would accept, in accordance with the indifference

[12] As just mentioned, we adopt the assumption that job destruction probabilities are the same during bargaining and production. Thus, since there is no intraperiod job destruction during production, we assume no job destruction between bargaining rounds within a period.

[13] When indifferent between accepting and declining, an agent is assumed to accept an offer.

condition to which we turn next). Symmetrically, in an even sub-period j, the equilibrium wage offer w_j^w extended by the worker would be such that the firm is indifferent between accepting the offer or waiting until the next sub-period to make the equilibrium counteroffer w_{j+1}^f:

$$(M-j+1)\left[y_\triangle - w_j^w\right] = -\gamma_\triangle + (M-j)\left[y_\triangle - w_{j+1}^f\right], \quad \text{for } j = 2, 4, \ldots, M-2, \tag{2}$$

where the left side is the firm's profits during the remainder of the period when accepting the wage offer, while the right side is the firm's value upon declining the offer, i.e., the incurrence of a cost of delay in bargaining $\gamma_\triangle$ and the subsequent earning of profits based on the firm's counteroffer (that the worker would accept, in accordance with indifference condition (1)).

After recursively substituting wages in equations (1) and (2), the first wage offer can be expressed as

$$w_1^f = \frac{z_\triangle + \left(\frac{M}{2} - 1\right)\left[z_\triangle + \gamma_\triangle + y_\triangle\right] + w_M^w}{M}. \tag{3}$$

Regarding the worker's take-it-or-leave-it offer w_M^w extended to the firm with a zero outside value, the worker would ask for the entire output $y_\triangle$ in that sub-period M and the expected present value of all future profits in the match,

$$w_M^w = y_\triangle + \sum_{i=1}^{\infty} \tilde{\beta}^i M \left(y_\triangle - w_1^f\right) = y_\triangle + \tilde{\beta} M \frac{y_\triangle - w_1^f}{1-\tilde{\beta}}, \tag{4}$$

where $\tilde{\beta} \equiv \beta(1-s)$, and β is workers' and firms' discount factor, and s is an exogenous job destruction probability between periods. After substituting (4) into (3), the first wage offer can be expressed solely in terms of primitives,

$$w_1^f = y_\triangle - \frac{1}{2}(1-\tilde{\beta})\left[y_\triangle - z_\triangle - \frac{M-2}{M}\gamma_\triangle\right]. \tag{5}$$

This is the wage that a firm would immediately offer a worker when first matched; the offer would be accepted. Hence, a firm's value of a filled job becomes

$$J = \sum_{i=0}^{\infty} \tilde{\beta}^i (y - w) = \frac{y - M w_1^f}{1-\tilde{\beta}} = \frac{1}{2}\left[y - z - \frac{M-2}{M}\gamma\right], \tag{6}$$

where we have substituted $M w_1^f$ for the per-period wage rate w and expression (5) for w_1^f.

In an equilibrium, the zero-profit condition in vacancy creation must hold,

$$c = \beta\, q(\theta)\, [J - \kappa]\,, \tag{7}$$

where the left side is the cost of posting a vacancy c, and the right side is the firm's expected discounted payoff. The probability $q(\theta)$ that a vacancy encounters an unemployed worker is a function of equilibrium market tightness θ: $q(\theta) = A\theta^{-\alpha}$ where $A > 0$ and $\alpha \in (0, 1)$. Upon encountering a worker, the firm pays the fixed cost κ before bargaining. Thus, the firm's payoff from an encounter is the value of a filled job J minus the fixed cost κ. After substituting expression (6) into (7), an equilibrium condition for market tightness is

$$\frac{c}{\beta q(\theta)} = \frac{1}{2}\left[y - z - \frac{M-2}{M}\,\gamma - 2\kappa\right]. \tag{8}$$

As in Ljungqvist and Sargent (2017), we can use implicit differentiation to compute the elasticity of market tightness with respect to productivity:

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;\frac{y}{y - z - \frac{M-2}{M}\,\gamma - 2\kappa} \;\equiv\; \Upsilon^{\text{sticky}}\;\frac{y}{y - z - \frac{M-2}{M}\,\gamma - 2\kappa}\,, \tag{9}$$

where the second factor is the inverse of the fundamental surplus fraction. Recall that the fundamental surplus is the difference between productivity y and model-specific quantities that the *invisible hand* cannot allocate to vacancy creation. In addition to the value of leisure z deducted in a standard Nash bargaining model, deductions now include a firm's cost γ of delay in bargaining. This item captures the worker's prospective gains from exploiting the cost that delay imposes on the firm. The third deduction is twice the value of the fixed cost κ that a firm must pay before bargaining with a newly-found worker.[14] We will return to the coefficient on κ below.

In AOB models, the inverse of the fundamental surplus fraction naturally governs how sensitively unemployment responds to productivity. Recall how zero-profit condition (7) in vacancy creation ensures that firms expect to break even when they post vacancies. A consequence is that a firm's gain from matching with an unemployed worker $(J - \kappa)$ will on

[14]For an equilibrium to exist, the fundamental surplus in the denominator of the second factor in decomposition (9) must be positive, i.e., parameters must satisfy

$$y > z + \frac{M-2}{M}\,\gamma + 2\kappa\,.$$

average have been spent on the vacancy posting costs incurred to fill that job. Interestingly, the firm's gain on the right side of expression (8) is exactly half of the fundamental surplus in the denominator of the second factor in decomposition (9). That constant fraction is the heart of the matter. First, since the fundamental surplus $y - x$ is a part of productivity y, it follows that a given percentage change in productivity translates into a larger percentage change in the fundamental surplus via the multiplier $y/(y - x)$, i.e., the inverse of the fundamental surplus fraction. Second, a consequence of the firm on average spending half of the fundamental surplus on filling a job is that a small fundamental surplus fraction magnifies the effect of a productivity change on the equilibrium quantity of resources spent on vacancy creation. Third, that magnified change on vacancy creation causes a large change in market tightness, as conveyed by decomposition (9), and hence, unemployment responds sensitively to small changes in productivity.[15]

To shed further light on the fundamental surplus in decomposition (9), we compare it to that of CET's Nash bargaining model based on the decomposition of market tightness elasticity specified in Section 2.2 and used in our Table 1,

$$\eta^{\text{Nash}}_{\theta,y} \;=\; \Upsilon^{\text{alt}}(\cdot)\;\frac{y}{y - z - \dfrac{1}{1-\phi}(1-\tilde{\beta})\,\kappa}\,, \tag{10}$$

where $1-\phi$ is the firm's Nash bargaining weight, and the factor $\Upsilon^{\text{alt}}(\cdot)$ satisfies a relatively low upper bound that has emerged from a professional consensus about values of the exogenous elasticity of matching with respect to unemployment.[16] The fundamental surplus for the

[15] CET (this issue) propose an alternative decomposition of the elasticity of market tightness that we reproduce here in the first equality of

$$\eta_{\theta,y} = \frac{1}{\alpha}\,\underbrace{\frac{y}{y - w - (1-(1-s)\beta)\kappa}}_{1/(\text{Profit rate})}\,\underbrace{\left[1 - \frac{dw}{dy}\right]}_{\text{Wage inertia}} = \frac{1}{\alpha}\,\underbrace{\frac{y}{\frac{1}{2}(1-\tilde{\beta})\left[y - z - \frac{M-2}{M}\,\gamma - 2\kappa\right]}}_{1/(\text{Profit rate})}\,\underbrace{\frac{1-\tilde{\beta}}{2}}_{\text{Wage inertia}} \qquad (\star)$$

where we use our notation for variables and parameters (as detailed in footnote 9). The second equality invokes equilibrium expression (5) for the wage rate $w = Mw_1^f$ and computes its derivative with respect to productivity y. Evidently, under the simplifying assumption of equal job destruction probabilities under bargaining and production, the elasticity is a function only of parameters.

To illustrate the limitations of CET's exclusive focus on the wage inertia term, consider two economies with different values of wage inertia, $0.5(1-\tilde{\beta})$, and ask which one of those economies has a higher elasticity of market tightness. Simplifying expression $(\star)$ to become our decomposition in (9) shows that the economy with the smaller fundamental surplus fraction has a higher elasticity of market tightness.

[16] According to the characterization in footnote 8, expression (10) can refer to either CET's Nash bargaining model or Pissarides's (2009) model analyzed by Ljungqvist and Sargent (2017, online appendix A.5) because only their first factor $\Upsilon^{\text{alt}}(\cdot)$ differ. Both first factors are bounded from above by $\max\{\alpha^{-1}, (1-\alpha)^{-1}\}$.

Nash bargaining model in expression (10) shares the same deduction with a unitary coefficient on the value of leisure z; but there is of course no cost γ for delay in bargaining as compared to the fundamental surplus for the AOB model in expression (9). As for the deduction associated with the fixed matching cost κ, let us sort out the difference in coefficients. In the contexts of layoff costs and fixed matching costs in the Nash bargaining framework, Ljungqvist and Sargent (2017) interpret associated deductions in fundamental surpluses as entailing annuitised values of those costs; here that corresponds to $(1-\tilde{\beta})\kappa$ in expression (10).[17] The other factor that makes up the coefficient on κ in expression (10) is the inverse of a firm's bargaining weight, $(1-\phi)^{-1}$, which arises because of the timing of that cost as explained in footnote 2. Given that the parameter ϕ (a worker's bargaining weight) is typically calibrated to lie in the mid-range of the unit interval, it follows that the latter factor, $(1-\phi)^{-1}$, would then take on values of around 2 and hence be of equal magnitude to that of the total coefficient on κ in expression (9). From that perspective, the difference between the deductions associated with κ in expressions (10) and (9) is that the annuitised value show up in the Nash bargaining model but the full value of κ in the AOB model. Why? The answer lies in CET's assumption that bargaining takes place within one model period over a finite maximum number of bargaining rounds, at the end of which, if bargaining runs its full course, the last wage offer will be extended by the worker as a take-it-or-leave-it offer to the firm. Consequently, CET's equilibrium bargaining outcome is one in which the firm is *essentially* confined to a single model period to recover its costs of forming a match, including the fixed cost κ. Therefore, the capital value κ rather than an annuitised value shapes the κ term in CET's AOB model.

5 Pleasant fundamental surplus arithmetic for CET

CET (this issue) confirm that our approximating version of the AOB model in the preceding section is the same as their AOB structure under an assumption of equal job destruction probabilities during bargaining and production ($\delta = 0$). From the perspective of the fundamental surplus, it is important that consensus then prevails on our decomposition (9) of the elasticity of market tightness, which is also reproduced by CET. While strictly speaking our formula (9) only applies to the far-right-end curvature of Figure 2 along which $\delta = 0$, we

[17] Under Nash bargaining, the fixed matching cost can be thought of as being amortized over the expected duration of a match. Let ψ be an annuity that, when paid for the duration of a match, has the same expected present value as a firm's fixed matching cost κ, so that $\sum_{i=0}^{\infty} \beta^i (1-s)^i \psi = \kappa \;\; \Rightarrow \;\; \psi = (1-\tilde{\beta})\kappa$.

can show that it is informative for the entire graph. Formula (9) is the multi-dimensional representation of a family of graphs like that of Figure 2 mapping out the importance of parameters z, γ and κ in the determination of how sensitively unemployment responds to productivity in CET's AOB model. Furthermore, formula (9) explains the tight negative linear relationship between γ and z that Papetti (2019) uncovers in his suite of restricted re-estimations of CET's AOB model, conditional on different values of γ. Since for producing a particular value of the elasticity of market tightness in formula (9) the two parameters are linearly related, variations in the postulated firms' cost of delay in bargaining γ are offset by estimated changes in the value of unemployment compensation z.

What implications does our simplifying $\delta = 0$ assumption have more generally for CET's AOB model? As Hall and Milgrom (2008) pointed out, the reason for high unemployment volatility in the AOB framework is the limited influence of unemployment (the outside value of workers) on wage outcomes. Hence, any increase in the probability δ that a job opportunity evaporates between bargaining rounds and sends the worker back to the unemployment pool will weaken the enhancement mechanism provided by the AOB framework by increasing the influence of a worker's outside value on wages. In Figure 2 this manifests itself as a widening of the lowlands at higher values of δ, so that it requires higher values of γ before reaching the steep slopes of the standard deviation of unemployment as a function of γ. From the perspective of the fundamental surplus, δ acts like a nuisance parameter that obscures the authentic determinants of how sensitively unemployment responds to productivity, i.e., the parameters z, γ and κ in the second factor of formula (9) for the approximating version. Admittedly, at small enough fundamental surplus fractions, perturbations of δ toward smaller values will significantly affect and increase what would already be a relatively high standard deviation of unemployment in Figure 2. Using the general decomposition of CET (2016, online appendix) that allows any value of δ, this would appear as a further increase in an already high second factor, i.e., an already small fundamental surplus fraction would become even smaller. But once again, this would obscure the fact that the authentic determinants of the elasticity of market tightness in the general decomposition would continue to be the parameters z, γ and κ: unless their combined values are large enough, the economy would be located far within the lowlands of Figure 2, and the value of δ would not matter much.[18]

[18]In their critique of the approximating version ($\delta = 0$), CET (2020) evaluate formula (9) at the parameter values of their estimated AOB model and find a negative elasticity. That negative number indicates that no equilibrium exists since parameters fail to satisfy the restriction presented in footnote 14. CET (this issue) acknowledge that non-existence and then proceed to use it as a reason for dismissing the approximating version. We prefer to emphasize the approximating version's strength as a tool for revealing the parameters

As evidence of the parameter δ not being central in CET's AOB analysis, the appendix sets $\delta = 0$ and recalibrates a critical parameter to attain the same standard deviation of unemployment as in CET's estimated AOB model. Our perturbed model generates virtually identical impulse-response functions with respect to 12 variables and three types of shocks as those computed by CET (2016) for the estimated model. Specifically, after setting $\delta = 0$, the recalibration is accomplished by reducing CET's parameter value of a firm's cost γ of delay in bargaining. This can be understood by looking at our Figure 2. As we walk back the value of δ from the estimate of 0.002 to zero, we move across upwards-sloping terrain. Hence, we need to lower the value of γ to stay at the same altitude – the same standard deviation of unemployment – as at the original point denoted 'CET' that refers to the estimated model in the figure. Furthermore, as in the construction of Figure 2, we adjust a firm's cost of posting a vacancy to make the average unemployment rate stay at 5.5 percent. Remarkably, by thus only targeting the standard deviation of unemployment in our perturbed model with $\delta = 0$, we can reproduce all the impulse-response functions for CET's estimated AOB model. This is yet another example of how to put the power of the fundamental surplus to work by confirming the explanatory power of the fundamental surplus when it classifies the parameter δ as a side-show in the context of identifying critical parameters that determine how sensitively unemployment responds to productivity. Furthermore, note that our alternative parameterization here is an example of a configuration that we call an *approximating version* of the model.

The simple decomposition in (9) derived from the approximating version, and the general decomposition of CET (2016, online appendix) each have strengths. The simple decomposition is enlightening about how to choose between candidates for a general decomposition, and its transparent revelation of critical parameters and forces of the fundamental surplus channel. First, a strength of the simple decomposition is that a consensus prevails about its derivation under its simplifying assumption of equal job destruction probabilities during bargaining and production spells. While a good guiding principle might seem to be to find a general decomposition cast in the image of formula (9), how to accomplish this is not obvious, to which the present exchange of views with CET testifies. As detailed in Section 2.2, the important issue here is whether the chosen general decomposition captures the essence of the decompositions of Ljungqvist and Sargent (2017), including our new formula (9), in

that determine the elasticity of market tightness in CET's AOB model. Thus, looking at Figure 2, while CET walk up along the far-right-end curvature where $\delta = 0$ and fall off a precipice at which no equilibrium exists, our panoramic view of the surface instead indicates how curvature at the far-right-end offers an informative overall topography of the relevant mountain walls and lowlands.

which the first factor is subject to a low upper bound and hence, the effects of different parameterizations on how sensitively unemployment respond to productivity are transmitted through changes in the second factor – the inverse of the fundamental surplus fraction. Our Section 3 observations confirm the virtues of the selected general decomposition by CET (2016, online appendix) while the alternative decomposition for the AOB model on the first line of CET's (this issue) Table 2 in which its first factor is evidently not constrained by any low upper bound utterly fails to shed light on the question under study here. Heuristically, the forces exerted by the critical parameters z, γ and κ identified by the approximating version are "seeping" into the first factor in that alternative decomposition and hence, no inference can be drawn from such a mixed-up and misleading decomposition. This insight from the approximating version and others discussed above demonstrate the strengths of the simple decomposition transparently to convey the essential components of the fundamental surplus channel. Furthermore, while implications from the general decomposition can be drawn out only numerically, the numerical studies we have read indicates that it performs as illustrated by, for example, Papetti's (2019) re-estimations of CET's AOB model.

6 Disarming the fundamental surplus channel

We can use CET's AOB model as an example to show how the fundamental surplus channel is disarmed when quantities deducted in the computation of the fundamental surplus covary procyclically with productivity. Let I^j, for $j \in \{z, \gamma, \kappa\}$, be an indicator equal to 0 if quantity j is either parametrically given or equal to 1 if quantity j is a fraction of productivity y, as given by $\bar{j} \cdot y$ where $\bar{j} \in (0, 1)$. Under these assumptions, the zero-profit condition (8) becomes adjusted to

$$\begin{aligned} \frac{c}{\beta q(\theta)} &= \frac{1}{2}\Bigg[\left(1 - I^z\bar{z} - I^\gamma \frac{M-2}{M}\bar{\gamma} - I^\kappa \cdot 2\bar{\kappa}\right) y \\ &\qquad -(1-I^z)z - (1-I^\gamma)\frac{M-2}{M}\gamma - (1-I^\kappa)\cdot 2\kappa\Bigg]. \end{aligned} \tag{11}$$

We can use implicit differentiation to compute the elasticity of market tightness with respect to productivity $\bar{\eta}_{\theta,y}$. To make it comparable to earlier elasticity reported in (9), we evaluate

the expression at the steady-state calibration $\bar{z}y = z$, $\bar{\gamma}y = \gamma$ and $\bar{\kappa}y = \kappa$,[19]

$$\bar{\eta}_{\theta,y} \;=\; \Upsilon^{\text{sticky}}\,\frac{y - I^z z - I^\gamma\,\dfrac{M-2}{M}\,\gamma - I^\kappa \cdot 2\kappa}{y - z - \dfrac{M-2}{M}\,\gamma - 2\kappa}\,. \tag{12}$$

We can verify that if no deductions vary with productivity, i.e., for $I^z = I^\gamma = I^\kappa = 0$, the elasticity $\bar{\eta}_{\theta,y}$ is identical to the earlier elasticity $\eta_{\theta,y}$ in (9). For each deduction j that is assumed to vary with productivity, setting $I^j = 1$ serves to suppress the elasticity $\bar{\eta}_{\theta,y}$. By setting $I^z = I^\gamma = I^\kappa = 1$, the smallest elasticity is attained at $\bar{\eta}_{\theta,y} = \Upsilon^{\text{sticky}}$, an outcome that completely disarms the fundamental surplus channel and that was anticipated by Ljungqvist and Sargent (2017, p. 2664, footnote 28):

> Extending a challenge that Chodorow-Reich and Karabarbounis (2016) presented to the matching literature, suppose that movements in productivity are associated with offsetting comovements in factors that affect deductions from productivity in the fundamental surplus. By arresting the fundamental surplus fraction, those offsetting changes would make unemployment unresponsive to productivity changes. While Chodorow-Reich and Karabarbounis investigated only the consequences of a procyclical value of leisure, similar consequences would flow from procyclicality in, e.g., fixed matching costs, a firm's cost of delay in alternating-offer wage bargaining, ...

In addition to setting $I^z = I^\gamma = I^\kappa = 1$, suppose that the cost of posting a vacancy c also varies with productivity, as given by $\bar{c}\,y$ where $\bar{c} > 0$. Zero-profit condition (11) then becomes

$$\frac{\bar{c}\,y}{\beta q(\theta)} = \frac{1}{2}\left(1 - \bar{z} - \frac{M-2}{M}\,\bar{\gamma} - 2\bar{\kappa}\right) y. \tag{13}$$

[19] Implicit differentiation of (11) yields

$$\frac{d\,\theta}{d\,y} = -\frac{\dfrac{1}{2}\left(1 - I^z\bar{z} - I^\gamma\,\dfrac{M-2}{M}\,\bar{\gamma} - I^\kappa \cdot 2\bar{\kappa}\right)}{-\dfrac{-q'(\theta)\,c}{q(\theta)^2\,\beta}} \;=\; \frac{\dfrac{c}{\beta q(\theta)}\;\left(1 - I^z\bar{z} - I^\gamma\,\dfrac{M-2}{M}\,\bar{\gamma} - I^\kappa \cdot 2\bar{\kappa}\right)}{\dfrac{\alpha\,c}{\theta\,q(\theta)\,\beta}\;\left(y - z - \dfrac{M-2}{M}\,\gamma - 2\kappa\right)},$$

where the second equality is obtained after using expression (11) to eliminate the quantity $\frac{1}{2}$ in the numerator, while in the denominator, we invoke the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$. Note that in substituting for $\frac{1}{2}$, we use the steady-state calibration $\bar{z}y = z$, $\bar{\gamma}y = \gamma$ and $\bar{\kappa}y = \kappa$ so that the *value* of expression (11) becomes identical to that of (8) and so that the two can be interchanged. Furthermore, that calibration is used again when computing the elasticity in (12).

Since y cancels in equation (13), market tightness θ does not depend on productivity; hence, the elasticity of market tightness with respect to productivity is zero. This is the route taken by Kehoe et al. (2019) who correctly claim to "have abstracted from the standard mechanism of *differential productivity across sectors* [market production versus home production and vacancy creation]." Indeed, Kehoe et al. confirm that under standard preferences with constant relative risk aversion, unemployment is unresponsive to productivity in their model, so alternative "exotic preferences" are required to explain volatility. Thus, even here the fundamental surplus serves as a handy tool for understanding ingredients needed to arrest "the standard mechanism of *differential productivity across sectors*" in matching models.

7 Concluding remarks

The fundamental surplus is a diagnostic tool that yields insights about the parameters that make unemployment respond sensitively to productivity across a diversity of matching models, including CET's.

These matching models adopt diverse structures that include sticky wages, elevated utility of leisure, bargaining protocols that suppress the influence of outside values, frictional credit market that gives rise to a financial accelerator, fixed matching costs, and government policies like unemployment benefits and layoff costs (see Ljungqvist and Sargent (2017)). The fundamental surplus unifies understanding these disparate models by isolating a single channel through which the economic forces that can generate a high elasticity of market tightness with respect to productivity must operate. In a nutshell, in order for unemployment to respond sensitively to productivity, the assorted deductions that lead to the fundamental surplus must be large enough to make the fundamental surplus fraction small.

The capacity of the fundamental surplus to isolate parameters essential for a high elasticity of market tightness supplements characterizations of endogenous relationships that arise in particular models. As an illustration, we return to two decompositions of the elasticity of market tightness for CET's AOB model in footnote 15 and in expression (9), respectively, where the latter refers to the approximating version with $\delta = 0$:

$$\frac{1}{\alpha}\ \underbrace{\frac{y}{y-w-(1-(1-s)\beta)\kappa}}_{1/(\text{Profit rate})}\ \underbrace{\left[1-\frac{dw}{dy}\right]}_{\text{Wage inertia}}\ =\ \eta_{\theta,y}\ =\Big|_{\delta=0}\ \ \frac{1}{\alpha}\ \frac{y}{y-z-\frac{M-2}{M}\gamma-2\kappa}\,. \tag{14}$$

It is interesting to learn how a high elasticity of market tightness depends on the endogenous

outcomes of a small profit rate and an inertial wage on the left side of expression (14). But what can guide us in calibrating the model in order to make the endogenous wage w and its derivative with respect to productivity dw/dy be sufficiently large and sufficiently small, respectively, to yield a high elasticity in this formula? The right side of expression (14) tells us the answer: we must set sufficiently large values of leisure z, a firm's cost γ of delay in bargaining, and the fixed cost κ that a firm must pay before bargaining with a newly-found worker.

Simulations and estimations of CET's AOB model confirm the presence of these parametric interdependencies. Furthermore, using the decomposition of market tightness elasticity derived by CET (2016, online appendix), confirms that the fundamental surplus fraction is inversely related to the simulated standard deviation of unemployment across diverse parameter configurations, such as in Figure 2. Remarkably, in Papetti's (2019) restricted estimations of CET's AOB model, the fundamental surplus fraction is virtually constant and small along a "ridge" of critical parameters that can explain observed unemployment volatility well. It is hard to imagine the parametric interdependencies that these restricted estimations reveal, a tell tale sign of diverse forces being funneled through a common channel – the fundamental surplus.[20] Knowing about the fundamental surplus prevents us from mistakenly downplaying any of the critical parameters such as the value of leisure that contribute to making unemployment respond sensitively to productivity. The fundamental surplus tells us that what matters are *combined* impacts of the critical parameters on the size of the fundamental surplus fraction. The fundamental surplus can also protect us from being misled to think that other parameters are critical when they are not. Thus, the fundamental surplus tells us that differences in job destruction probability during bargaining versus production are a side-show.

[20]For another example of parametric interdependencies in a more complicated environment, as mentioned in Section 2.1, we also studied an application on the consequences of alternative welfare state arrangements in Ljungqvist and Sargent (2017, sec. VII.B). Namely, Hornstein, Krusell, and Violante (2007) incorporate matching in the labor market of a vintage-capital growth model, and argue that the increase in European unemployment since the 1980s can be attributed to a higher rate of capital-embodied technological change. They also conclude that it is the combined effects of welfare state institutions that matter and "reforming any one institution could reduce dramatically the elasticity of the unemployment rate to obsolescence shocks." As we demonstrate, the latter conclusion is a manifestation of the intermediate channel of the fundamental surplus. Furthermore, upon inspection of the deductions that must be made to arrive at the fundamental surplus, we discover that the authors made a mistake when computing the new balanced-growth trajectories for the amounts of unemployment benefits and the layoff cost; inadvertently, making the replacement rate in unemployment insurance more generous and the layoff tax in terms of the wage rate more onerous. After correcting for that mistake, the model can no longer explain the increase in European unemployment.

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Appendix: Perturbation of CET's AOB model with $\delta = 0$

We perturb the estimated AOB model in Christiano, Eichenbaum, and Trabandt (2016) by setting $\delta = 0$, i.e., no intraperiod job destruction during bargaining, and recalibrating a firm's cost γ of delay in bargaining so as to target the same standard deviation of unemployment as in the estimated model equalling 0.197. We adjust a firm's cost c of posting a vacancy to make the average unemployment rate stay at 5.5 percent. Specifically, γ is decreased from 0.0090 to 0.0068, and c is raised from 0.0029 to 0.023. Impulse-response functions for 12 variables with respect to a neutral technology shock, an investment-specific technology shock, and a monetary policy shock are shown in Figures 3, 4 and 5, respectively. Note that there is almost a perfect overlap of these impulse-response functions and those computed by Christiano et al. (2016) for the estimated model.

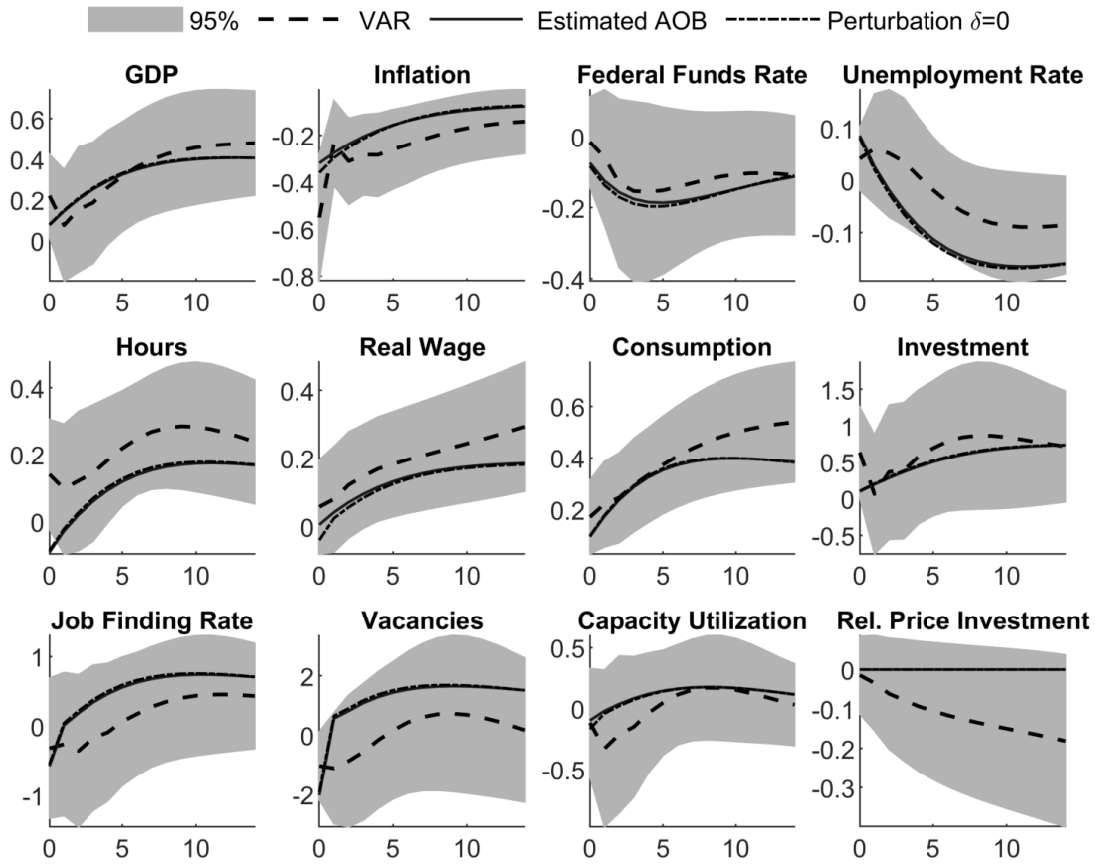

Figure 3: Impulse responses with respect to a neutral technology shock in CET's AOB model. The solid lines refer to the estimated model and the dashed-dotted lines to the perturbed model with $\delta = 0$. Note that there is an almost perfect overlap of the solid and the dashed-dotted lines.

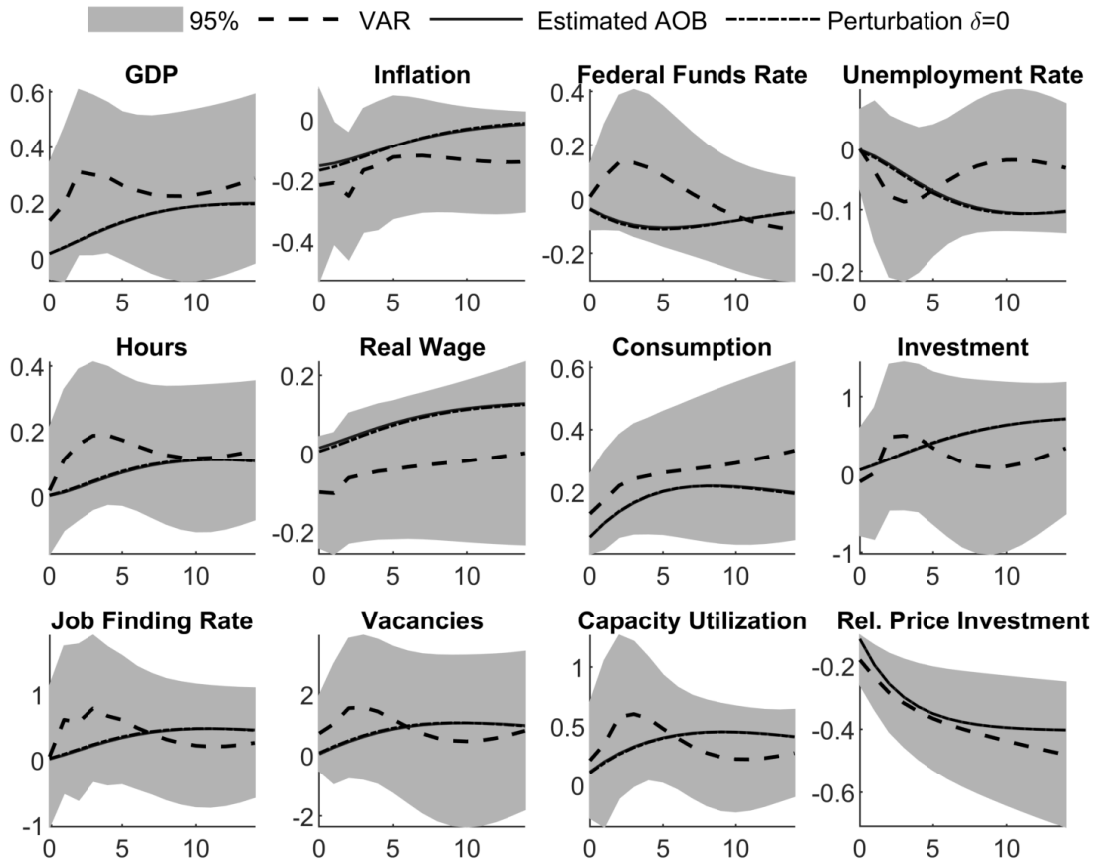

Figure 4: Impulse responses with respect to an investment-specific technology shock in CET's AOB model. The solid lines refer to the estimated model and the dashed-dotted lines to the perturbed model with $\delta = 0$. Note that there is an almost perfect overlap of the solid and the dashed-dotted lines.

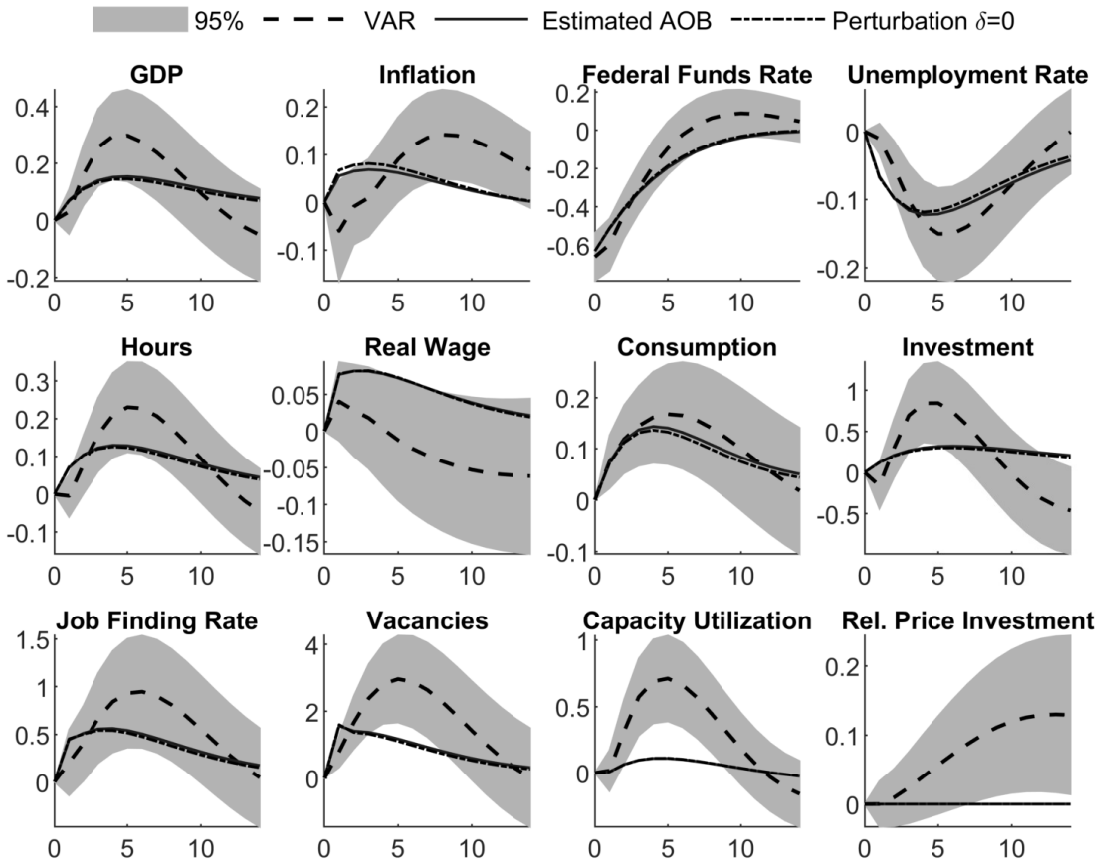

Figure 5: Impulse responses with respect to a monetary policy shock in CET's AOB model. The solid lines refer to the estimated model and the dashed-dotted lines to the perturbed model with $\delta = 0$. Note that there is an almost perfect overlap of the solid and the dashed-dotted lines.