HAOK and HANK Models

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Abstract

I compare Heterogeneous Agent Old Keynesian with Heterogeneous New Keynesian models. I describe evidence and data reduction techniques that convinced 20th century macroeconomists to embrace HAOK models. I describe other evidence that persuaded some 21st century macroeconomists to use HANK models. The HANK project carries vast macroeconomic policy consequences.

Keywords: Neoclassical synthesis, scandal, price level targeting, redistribution, functional vector autoregression, descriptive, structural

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1 Introduction

Accounting for and managing heterogeneities in economic agents' preferences, information sets, and opportunities have always been central to macroeconomic theory. Long before macroeconomics existed as a distinct field, conflicts of interest preoccupied those who designed monetary-fiscal policies (see appendix A for some 19th century US examples). Section 2 describes Heterogeneous Agent Old Keynesian (HAOK) models and the reasons that distinguished twentieth century macroeconomists used them to analyze consequences of alternative monetary and fiscal policies. Section 3 describes how informal NBER reference cycle models created by Burns and Mitchell (1946), and single-factor descriptive statistical models like those sketched by Koopmans (1947), and formalized by Sargent and Sims (1977), framed evidence that motivated HAOK theorists. The goal of quantifying HAOK models motivated construction of a statistical theory for estimating systems of vector difference equations. Section 4 recalls Kenneth Arrow's skepticism about the consistency of HAOK models with modern general equilibrium theory. Section 5 describes how authors of HANK models challenge key empirical motivations underlying HAOK models and how they subvert logic underlying the light-handed fiscal-monetary policies affiliated with a Neo Classical Synthesis. Section 6 tells how functional autoregressions and related descriptive statistical models are being used to gather evidence that might discriminate between HAOK and HANK models. Section 7 concludes by offering opinions about how the HANK project creates promises and controversies.

2 The neoclassical synthesis

The K in HAOK and HANK honors John Maynard Keynes. It is useful to recall the sense in which he intended Keynes (1936) to be a general theory. Keynes wanted a theory that:

- Explains equilibria with underemployed resources and excess supplies.
- Reduces to "classical" (i.e., Walrasian) general equilibrium theory when resources are fully employed.
- Rationalizes light-handed fiscal-monetary interventions that depend only on aggregate data.

Keynes wanted macroeconomic policies that promote aggregate efficiency while letting individuals' choices guide the allocation of resources. To accomplish those goals he advocated

- A price level target.[1]
- Keeping two government budgets, a current account and a capital account:
  - Always balancing the current account budget.
  - Not requiring period-by-period balancing of the capital budget but requiring only its present-value balance.
  - Using countercyclical capital-account deficits, but not current-account deficits, to finance public works.[2]

Keynes's advocacy of these light-handed macroeconomic policies presumed the presence of a UK 1920's style social safety net.

In a nutshell, Keynes advocated (i) achieving full employment by using well timed public investment to sustain adequate demand, and then (ii) relying on markets to set relative prices and allocations. Paul Samuelson called this theory-policy package a "Neoclassical Synthesis." Here is how Keynes described it:

> When 9,000,000 men are employed out of 10,000,000 willing and able to work, there is no evidence that the labour of these 9,000,000 men is misdirected. The complaint against the present system is not that these 9,000,000 men ought to be employed on different tasks, but that tasks should be available for the remaining 1,000,000 men. It is in determining the volume, not the direction, of actual employment that the existing system has broken down.
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> J. M. Keynes, *General Theory*, 1936, chapter 24.

A package of ideas that culminated in his neoclassical synthesis emerged gradually during the years from 1911 to 1931 when Keynes practiced what he later called "classical" macroeconomics. To follow his progress, read (Keynes, 1924, ch. 1) in which he analyzed how inflation disrupted (1) distributions of wealth and consumption among (a) investors, (b) the business class, and (c) earners as well as (2) production (i.e., the allocation of resources).[3] His analysis of those disruptions led Keynes to advocate price level targeting:

[1] Keynes (1924, 1925) emphasized the priority of present value government budget balance as essential determinant of the price level.

[2] Proposals to time public works to attenuate the business cycle were in the air in the 1920s. For example, see Foster et al. (1928) and Foster and Catchings (1930).

[3] The way Keynes (1924, ch. 1) sorted through effects of inflation on distribution and production reminds me of recent analyses of contending effects of alternative government policies in HANK models in terms of imputations of welfare consequences of alternative government policies that flow from (i) redistribution, (ii) insurance, and (iii) efficiency. See Bhandari et al. (2023, 2021).

> We leave Saving to the private investor, and we encourage him to place his savings mainly in titles to money. We leave responsibility for setting Production in motion to the business man, who is mainly influenced by the profits which he expects to accrue to himself in terms of money. Those who are not in favor of drastic changes in the existing organization of society believe that these arrangements, being in accord with human nature, have great advantages. But they cannot work properly if the money, which they assume as a stable measuring-rod, is undependable. Unemployment, the precarious life of the worker, the disappointment of expectation, the sudden loss of savings, the excessive windfalls to individuals – the speculator, the profiteer – all proceed, in large measure, from the instability of the standard of value.
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> John Maynard Keynes, *A Tract on Monetary Reform*, 1924.

Keynes disapproved of episodes of redistributions via unforeseen inflations:

> There is no record of a prolonged war or a great social upheaval which has not been accompanied by a change in the legal tender, but an almost unbroken chronicle in every country which has a history, back to the earliest dawn of economic record, of a progressive deterioration in the real value of the successive legal tenders which have represented money.
>
> John Maynard Keynes, *A Tract on Monetary Reform*, 1924.

He regarded those past inflation-engineered redistributions as purposeful:

> Moreover, this progressive deterioration in the value of money through history is not an accident, and has had behind it two great driving forces – the impecuniosity of Governments and the superior influence of the debtor class.
>
> ... the benefits of a depreciating currency are not restricted to the Government. Farmers and debtors and all persons liable to pay fixed money dues share in the advantage. As now in the persons of business men, so also in former ages these classes constituted the active and constructive elements in the economic scheme.
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> John Maynard Keynes, *A Tract on Monetary Reform*, 1924.

Appendix A provides some US historical examples of the episodes that Keynes probably had in mind.[4] The appendix describes some nineteenth-century controversies about how the US Federal government should use fiscal-monetary policy to redistribute wealth among nominal net creditors and debtors, controversies that recurred often from the founding of the US republic until Keynes's time. Instances of the same controversies occurred in England, France, and other European countries in the 18th and 19th centuries. Keynes participated actively and passionately in widespread debates about similar issues that occurred in Europe after World War I. Keynes's response to these debates was to advocate separating a government's price-level goals from its concerns about redistribution:[5]

Keynes advocated targeting the price level.

> If we are to continue to draw the voluntary savings of the community into "investments," we must make it a prime object of deliberate State policy that the standard of value, in terms of which they are expressed, should be kept stable; adjusting in other ways (calculated to touch all forms of wealth equally and not concentrated on the relatively helpless "investors") the redistribution of the national wealth, if in the course of time, the laws of inheritance and the rate of accumulation have drained too great a proportion of the income of the active classes into the spending control of the inactive.
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> John Maynard Keynes, *A Tract on Monetary Reform*, 1924.

Samuelson, Tobin, Friedman, Lucas, Prescott, and other creators and practitioners of 20th century macroeconomics accepted and implemented Keynes's "neo-classical synthesis". But first they had to resolve the ambiguities and confusions inherent in Keynes's mostly literary (i.e., non-mathematical) style of analysis. A project to do that began with a string of contributions by Hicks (1937), Tinbergen (1939), Samuelson (1939), Modigliani (1944), and Tobin (1955). They translated and transformed Keynes's analysis into a "general equilibrium" system of n-equations in n unknowns having a neat partition into n endogenous variables and several exogenous variables representing monetary and fiscal policy actions. Solutions of those equations could be used to analyze alternative settings of the government monetary and fiscal actions. To perform the types of statistical implementation and verification of Keynes's "General Theory" that Tinbergen sought, it was necessary to have in

[4]Brunnermeier et al. (2023) document how German monetary policy during the 1922-1923 hyperinflation purposefully benefitted some citizens at the expense of others. See Newcomb (1865) for a related analysis and criticism of US monetary policy during the 1861-1865 Civil War.

[5]Also see Keynes (1931a).

hand a specific "n-equations in n unknowns" system of this kind. All of these early works accepted Keynes's reasoning in terms of broad macroeconomic aggregates – employment, interest, and money – because the Great Depression of the 1930's convinced them that understanding and attenuating adverse fluctuations in those aggregates were scientific problems of pressing moral importance.

Meanwhile, little impressed or influenced by Keynes's theorizing but vitally interested in business cycles, for many years Wesley C. Mitchell and Arthur Burns and their teammates at the National Bureau of Economic Research had patiently interrogated many "witnesses" to US business cycles by assembling and studying time series of a diverse collection of quantities and prices, a long line of work that culminated in Burns and Mitchell (1946). From an immense data set they extracted a US business cycle by using a home-made data-reduction technique. To summarize their data set, they constructed a 9-part "reference cycle" onto which they "projected" each of their many time series. From their inductive approach, they organized evidence that, even to economists having more taste and patience for economic theory than Burns and Mitchell did, seemed to justify a constructing macroeconomic theory.

Although Burns and Mitchell (1946) and Tinbergen (1939) used very different methods, both were interested in the same data that somehow "nature" had generated by one process. Both sought to learn about that process by enlisting what modern statisticians call an "inductive bias" or "statistical prior". Indeed, both hypothesized a single-dimensional aggregate. Filling in technical details required to justify and extend the analytical approach of either Burns and Mitchell (1946) or Tinbergen (1939) would require talent and time. Thus, the statistical theory appropriate for estimating parameters of a system of n equations in n unknowns – required to complete Tinbergen's project – had not yet been created. Connections between such a statistical theory and the sorts of statistics that Burns and Mitchell (1946) had assembled were unknown.[6] In the next section, we briefly describe early efforts to learn these connections.

3 Two types of statistical model

In the tradition of Koopmans (1950), I define a statistical model as a probability distribution $f(y|\theta)$ of a random vector y indexed by parameters $\theta \in \Theta$. The set Θ describes a manifold of statistical models. In economics and other sciences too, statistical models come, or pretend to come, in two types, descriptive and structural.

[6] King and Plosser (1994) connect the two approaches.

- Parameters θ_{descr} of a descriptive model are data summarizers like regression coefficients and entries of covariance matrices of shock vectors. These parameters are not directly interpreted as preference or technology parameters of an economic theory. Instead, they are dimension-reducers, i.e., data-compression devices.
- Some or all of the parameters θ_{struct} of a structural model pin down preferences, technologies, endowments, information structures, surprises that instigate 'mistakes of foresight', and so on. These parameters are objects in which economic theories are cast.

Descriptive models are designed to detect patterns and assemble interesting "facts", but not to explain them. Structural models are designed to explain them in terms of the parameters that quantify determinants of demands and supplies. Both types of model play important roles in macroeconomics. The purposes of a descriptive model are dimension reduction, data compression, and pattern recognition. The purpose of a structural model is to uncover invariants that can support theoretical analysis of historically unprecedented policy interventions.

Koopmans and his colleagues at the Cowles Commission initiated a research program that would connect the two types of statistical models. Koopmans (1947) wanted to construct a mapping $\theta_{descr} = F(\theta_{struct})$ so that he could study how to invert it and recover $\theta_{struct} = F^{-1}(\theta_{descr})$. Koopmans (1949, 1950) advocated "structural" Keynesian econometric models that could be used to recommend aggregate demand management policies that would implement the neoclassical synthesis.[7, 8]

Mid twentieth century theorists and econometricians who were inspired by the noble goal of understanding and moderating business cycles and preventing a recurrence of that geopolitical disaster, Great Depression of the early 1930s, introduced a distinction between descriptive and structural statistical models that pervades applied econometrics today.[9] Leading theorists and econometricians repaired loose ends left by Keynes by representing his ideas as

[7]Koopmans (1949, 1950) usually started with a structural model with parameters θ_{struct} and then deduced an associated "reduced form" descriptive model with parameters $\theta_{descr} = G(\theta_{struct})$. A major theme of Hansen and Sargent (2013) was to pursue this approach by characterizing the mapping from a structural dynamic model that takes the form of a linear hidden Markov model to an associated vector autoregression that characterizes its likelihood function and that represents its reduced form. Unfortunately, today "reduced form" is too often used, not in its original Cowles Commission sense, but in the corrupted sense of "incompletely articulated descriptive model".

[8]Koopmans prefigures what we now call Indirect Inference as perfected by Gallant and Tauchen (1996). For Gallant and Tauchen an auxiliary model is a descriptive statistical model that (1) is a likelihood function that describes data well, and (2) can be computed and maximized easily. It is a good idea to estimate structural model by using score functions of an auxiliary model to generate an appropriate Generalized Method of Moments criterion.

[9]It pervades "machine learning" as well.

n equations in n unknowns that formed vector stochastic difference equations that could be matched to data. In the five years after WWII, parallel efforts by raw empiricists Burns and Mitchell at the National Bureau of Economic Research and theorist-econometricians at the Cowles Commission, first at the University of Chicago and then at Yale, came to fruition. A memorable debate pitted Koopmans against Burns and Mitchell and posed enduring issues. Koopmans was remarkably even-handed in setting forth and refining a case for using Burns and Mitchell's approach before delineating its limitations:

> When Tycho Brahe and Johannes Kepler engaged in the systematic labor of measuring the positions of the planets, and charting their orbits, they started with conceptions and models of the planetary system which later proved incorrect in some aspects, irrelevant in others. Tycho always, and Kepler initially, believed in uniform circular motion as the natural basic principle underlying the course of celestial bodies. Tycho's main contribution was a systematic accumulation of careful measurements. Kepler's outstanding success was due to a willingness to strike out for new models and hypotheses if such were needed to account for the observations obtained. He was able to find simple empirical "laws" which were in accord with past observations and permitted the prediction of future observations. This achievement was a triumph for the approach in which large scale gathering, sifting, and scrutinizing of facts precedes, or proceeds independently of, the formulation of theories and their testing by further facts.
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> ... in due course, the theorist Newton was inspired to formulate the fundamental laws of attraction of matter, which contain the empirical regularities of planetary motion discovered by Kepler as direct and natural consequences. The terms "empirical regularities" and "fundamental laws" are used suggestively to describe the "Kepler stage" and the "Newton stage" of the development of celestial mechanics. It is not easy to specify precisely what is the difference between the two stages. Newton's law of gravitation can also be looked upon as describing an empirical regularity in the behavior of matter. The conviction that this "law" is in some sense more fundamental, and thus constitutes progress over the Kepler stage, is due, I believe, to its being at once more elementary and more general. It is more elementary in that a simple property of mere matter is postulated. As a result, it is more general in that it applies to all matter, whether assembled in planets, comets, sun or stars, or in terrestrial objects - thus explaining a much wider range of phenomena. Koopmans (1947, p. 161)

> even for the purpose of systematic and large scale observation of such a many-sided phenomenon, theoretical preconceptions about its nature cannot be dispensed with .... Koopmans (1947, p. 163)

As a sympathetic and constructive critic of Burns and Mitchell's reference-cycle technique, Koopmans indicated how it could be formalized as a single-factor dynamic version of a factor-analytic model of the type that psychologists had used to summarize student test scores as an intelligence quotient,[10]

> The notion of a reference cycle itself implies the assumption of an essentially one-dimensional basic pattern of cyclical fluctuation, a background pattern around which the movements of individual variables are arranged in a manner dependent on their specific nature as well as on accidental circumstances. (There is a similarity here with Spearman's psychological hypothesis of a single mental factor common to all abilities.) This "one-dimensional" hypothesis may be a good first approximation, in the same sense in which the assumption of circular motion provides a good first approximation to the orbits of the planets. It must be regarded, however, as an assumption of the "Kepler stage," based on observation of many series without reference to the underlying economic behavior of individuals. It is in this sense, I believe, that the authors refer (p. 3) to their definition of business cycles as "a tool of research, similar to many definitions used by observational sciences, and like its analogues subject to revision or abandonment if not borne out by observation." I believe that the authors would not object to the addition: "or by the logical consequences of observations of a wider range of phenomena." Koopmans (1947, p. 165)

Thus, Koopmans indicated that some of Burns and Mitchell's data summaries could be organized and sharpened in terms of a factor analytic model, a suggestion that Geweke (1977), Sargent and Sims (1977), and Geweke and Singleton (1981) and others would eventually pursue.

Although Koopmans (1947) had regarded *Measuring Business Cycles* by Burns and Mitchell (1946) as an extensive pattern-recognition and data reduction exercise that fell short of formally producing a descriptive statistical model, even without such a formalization, Burns and Mitchell's concept of a one-dimensional "reference cycle" influenced leading macroeconomic model builders. I audited Robert E. Lucas's economics 331 PhD first-year

[10]Lovie and Lovie (1993) describe origins and early applications of factor analysis.

macro class at the University of Chicago in the winter quarter of 1977. Lucas devoted several lectures to describing Burns and Mitchell's procedures for constructing reference cycles through a process of taking moving averages, removing trends, and applying subjective judgments. Using Brock and Mirman (1972) as a benchmark model, Lucas took Burns and Mitchell's single-factor "all business cycles are alike" finding as his starting point. Then he set out to explain "real" and "nominal" outcomes in terms of preferences and constraints facing households, firms, and governments. From Burns and Mitchell's diagrams and other sources, Lucas inferred that, while a one-factor model could approximate quantities well, it seemed that another factor was needed to account for nominal prices. Additional tentative support for Lucas's inferences emerged from Sargent and Sims (1977).

In summary, two interrelated ideas guided authors of HAOK models: (1) an empirical judgment that "all business cycles are similar" captured by Burns and Mitchell's application of their reference-cycle procedure to many US time series, and (2) Keynes's "neoclassical synthesis" that justified James Tobin's definition of macroeconomics as "a field that ignores distribution effects". While many leading US economists after World War II endorsed this approach, not everyone did.

4 Arrow's challenge

When he reviewed the collected works of Paul Samuelson (1966), Kenneth Arrow called the Neoclassical Synthesis a scandal:.

> . . . Samuelson has not addressed himself to one of the major scandals of current price theory, the relation between microeconomics and macroeconomics. Neoclassical macroeconomic equilibrium with fully flexible prices presents a beautiful picture of the mutual articulations of a complex structure, full employment being one of its major elements. What is the relation between this world and either the real world with its recurrent tendencies to unemployment of labor, and indeed of capital goods, or the Keynesian world of an underemployment equilibrium?
>
> Arrow (1967, p. 734)

Arrow asserted that

> If the neoclassical model with full price flexibility were sufficiently unrealistic that stable unemployment equilibrium be possible, then in all likelihood the bulk of the theorems derived by Samuelson, myself, and everyone else from the

> neoclassical assumptions are also counterfactual. The problem is not resolved by what Samuelson has called "the neoclassical synthesis," in which it is held that achievement of full employment requires Keynesian intervention but that neoclassical theory is valid when full employment is reached.
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> Arrow (1967, p. 735)

Elaborating, Arrow wrote

> The Samuelson-Keynes view of the world is that full employment is a valid proposition in $K(g)$ only for special values of g, whereas full employment holds in $W(g)$ for all g. If g^* is such that full employment holds in $K(g^*)$, can it be true that theorems valid in $W(g^*)$ are also valid in $K(g^*)$? Obviously, it is not true that the two systems respond similarly to changes in g, since full employment remains valid in one but not in the other.
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> Arrow (1967, p. 735)

It is natural to expect that Arrow's criticisms would be taken to heart especially by rational expectations macroeconomists like Lucas and Prescott who were eager to bring lessons from Arrow's and Debreu's analysis of general models into macroeconomics (see Prescott and Lucas (1972)). Lucas (1987) addressed some of Arrow's doubts, though at the end of the day, Lucas embraced the neoclassical synthesis. Manuelli and Sargent (1988) discussed some of the steps that Lucas took to separate redistribution and insurance from the determinants of aggregate outcomes.

After criticizing the theoretical foundations of the neoclassical synthesis, Arrow commended statistical findings that had modified recent refinements of macroeconomic theories:

> The major developments, the development of more subtle theories of the consumption function and the distributed-lag theories of investment, have been closely associated with econometric investigation.
>
> Arrow (1967, p. 733)

In section 3, we described empirical findings that fortified a HAOK modeling tradition that embraced a Neoclassical Synthesis. In section 6 we'll describe how more recent investigations bear on the HANK project.

5 HANK models

Although a neoclassical synthesis dominated quantitative macroeconomics for many decades, heterogeneous agent models were always present and taken seriously as early as the multiple-class models of Kalecki (2016) that emphasized heterogeneous marginal propensities to consume and their implications for fiscal policy. Indeed, important components of Friedman (1956) were his empirical and theoretical analyses of differences in marginal propensities to consume across classes of consumers who faced stochastic processes of non-financial income with different mixtures of permanent and temporary components. Furthermore, a substantial body of work by macroeconomists occupying the last third of Ljungqvist and Sargent (2018) applied recursive contracts to analyze how to arrange social insurance in the presence of information and enforcement difficulties.[11]

Nevertheless, n-equations-in-n-unknowns quantitative models of macroeconomic equilibrium continued to be cast in terms of macroeconomic aggregates (i.e., cross section averages).[12] Macroeconomists refined how to acknowledge heterogeneity but still preserve a macroeconomic analysis cast solely in terms of aggregates. Prominent examples include Lucas (1982, 1987, 2003).[13] Thus, recall how Lucas (1982) carefully arranged a complete set of state-contingent contracts and an initial distribution of wealth across countries to prevent the distribution of wealth across countries from affecting prices and aggregate quantities. Lucas (1987, 2003) assumed a complete set of state-contingent contracts, an effective social safety net, and a monetary-fiscal policy that eliminated avoidable adverse fluctuations. I read Lucas as estimating the residual gains to aggregate efficiency that remained possible beyond those that had been achieved by Volcker and Greenspan. His finding that they were small induced Lucas to advocate focusing research and policy improvements on secular growth, not on further attenuating business cycles. In similar ways, creators of representative agent

[11] Interesting examples of such work are Pavoni and Violante (2007) and Pavoni et al. (2016) who analyze optimal arrangements for inducing welfare recipients to enter gainful employment. They do "recursive mechanism design", also known as "dynamic programming squared", in which history dependent allocations are represented recursively by using agents' continuation values as state variables in a planner's value function. Thus, Pavoni et al. (2016) deploy "... several policy instruments (e.g., job-search, assisted search, mandated work) the principal can use, in combination with welfare benefits, in order to minimize the costs of delivering promised utility to the agent. The generosity of the program and the skill level of the unemployed agent determine the optimal policy instrument to be implemented."

[12] Edward Prescott urged his students and everyone else who would listen to say "aggregate economics", not "macroeconomics".

[13] Prescott (2006a, 2005, 2006b) used distinct theories of aggregation to construct an aggregate labor supply curve, one based on Rogerson employment lotteries, the other based on incomplete markets, self-insurance, and time-averaging. He switched from one to the other in between the two published versions of his Nobel lecture.

New Keynesian (RANK) models that swept into central banks and macro textbooks in the 1990s also pushed heterogeneity into the background to justify casting their n-equations-in-n-unknowns models in terms of aggregates.[14]

Then along came HANK models.

HANK models are part of a broad project to put heterogeneity front and center in macroeconomics. They substantially increase the dimension n in n-equation-in-n unknown models by including higher moments of cross-sections of wealth and income components as determinants of cross-section means. Dynamic programming, dynamic programming squared (i.e., recursive contracts), vector autoregressions, and structural macroeconometrics are HANK modelers' hammers and saws. The HANK revolution is not about tools but about substance. HANK research undermines the neoclassical synthesis in several ways. First, it contributes descriptive statistical models (for example see Guvenen et al. (2014), Guvenen et al. (2021), and Heathcote et al. (2024)). These models detect relations among the higher moments and the means of cross sections of incomes and wealth means. They indicate that current values of higher moments contain information about future cross section averages. Second, it has invented structural HANK models (for example, see Kaplan et al. (2018) and Kaplan and Violante (2018)) that undermine the HAOK prescription from Keynes that macroeconomic policy should be light-handed and separate from policies that redistribute income and wealth. Furthermore, HANK modelers would replace a low-inflation mandate (or a low-inflation plus low-unemployment mandate) for a Central Bank and focus instead on other outcomes.

Thus, Bhandari et al. (2021) apply recursive contracts analysis to an *ex ante* heterogeneous agent HANK model. They compare outcomes and policies under optimal history-dependent policies with those recommended by ordinary Taylor rule and interpret differences in terms of motivations of a Ramsey planner. Responses of optimal policies to aggregate shocks differ qualitatively from what they would be in a corresponding representative agent economy. They are an order of magnitude larger. An ordinary Taylor rule is strongly dominated. A motive to provide insurance that arises from heterogeneity and incomplete markets outweighs price stabilization motives that ordinarily rule in a representative agent New Keynesian model. To understand sources of welfare gains relative to an ordinary Taylor rule, they use a decomposition of those gains proposed by Bhandari et al. (2023) into parts attributable to insurance, redistribution, and aggregate efficiency. They find that an insurance component is positive and greater than 100%, that a redistribution component is small, and that an aggregate efficiency component is negative. They summarize their results as follows:

[14]For many RANK models, $n = 3$.

> ... essentially all the welfare gains from optimal HANK policies arise from the additional insurance that they provide. Provision of insurance comes at the cost of sacrificing price stability, which creates deadweight losses and lowers total aggregate resources available for consumption. This explains why the aggregate efficiency component is negative.

6 Functional autoregressions and HANK

The HANK modeling project fosters both descriptive and structural statistical models. In terms of descriptive models, new tools, or extensions of old ones, are being applied to revisit Burns and Mitchell's (1946) characterization of business cycles with NBER reference cycles and with dynamic versions of the Spearman single factor models mentioned by Koopmans (1947). This work is directed at reexamining and refining the single-factor characterization of macro time series that originally buttressed the neoclassical synthesis. Here I briefly describe a useful tool for constructing descriptive models of cross-section dynamics that extends the vector autoregression technology that for 45 years macroeconomists have deployed to construct descriptive models of macroeconomic variables. Its purpose is to construct an autoregression for a stochastic process of cross-section densities $p_t(x), t \in T$, where T is the set of integers. Density $p_t(x)$ has dimension infinity. It is convenient to work with log densities $\ell_t(x) = \log p_t(x)$ and to fit a VAR for an $\ell_t(x)$ process. To approximate an infinite dimensional VAR, one estimates a finite K-dimensional VAR for coefficients of K basis functions for a cross-section density. Thus, let a first-order functional VAR be

$$\ell_{t+1}(x) = \int B(x, \tilde{x}) \ell_t(\tilde{x}) d\tilde{x} + u_{t+1}(x)$$

or

$$\ell_{t+1} = B\ell_t + u_{t+1}, \quad u_{t+1} \perp \ell_t$$

Make an approximation

$$\ell_t(x) \approx [\xi_1(x), \ldots, \xi_K(x)] \cdot [\alpha_{1t} \ldots \alpha_{Kt}],$$

where the basis functions $\xi_i(x)$ might be sieves or functional principal components. Run a first-order VAR on the basis coefficients

$$\alpha_{t+1} = A\alpha_t + u_{\alpha,t+1}, \quad u_{\alpha,t+1} \perp \alpha_t$$

Then back out approximate log cross-section densities $\ell_t(x)$

Time series macroeconometricians at the University of Indiana have fit functional VARs to interesting cross section log densities. They have fit functional VARS as ingredients of both descriptive and structural statistical models. To acknowledge the prevalence of stochastic geometric growth in state-of-the-art ways, Chang et al. (2019) describe how to incorporate co-integration and additive functionals in the spirit of Hansen (2012). Liu and Plagborg-Møller (2021) estimate a heterogenous-agent structural model. Chang et al. (2022a) formulate a functional VAR for aggregates and a cross-section consumption density as a Hidden Markov Model. More Indiana macro is on the way in a work-in-progress paper Chang et al. (2022b).

Findings of these papers bear on the plausibility and promise of the HANK project. I'll confine myself here to a few remarks about Chang et al. (2022a). After they fit a descriptive functional VAR as a hidden Markov model, in the process displaying high technical virtuosity, they offer an informative discussion of mappings $\theta_{struct} = F^{-1}(\theta_{descr})$ for some HANK models simulated under some interesting scenarios. Their findings are bound to be controversial because their descriptive model detects limited dynamic influences that pass from higher cross-section moments to cross-section averages. This seems to be a discouraging finding for the HANK project. But I hesitate to conclude that because maybe the findings describe outcomes after prevailing social safety-net and aggregate demand management policies have generated effective "off-equilibrium" feedbacks from cross-section dynamics to aggregates, while observed equilibrium paths conceal those feedbacks. This interpretation is a counterpart to my earlier interpretation of costs of business cycles quantified by Lucas (2003).

7 Concluding remarks

The HANK project is promising and provocative. It is being pursued by technically able researchers who are full of ideas and analytical powers, and who thoroughly know the HAOK and real business cycle models that they want to improve.[15] Their HANK project has an electric charge and is bound to be controversial because it challenges the neoclassical synthesis and a widely-believed prescription for separating macro policy design from policies to redistribute income and wealth. Because they undermine single- and dual-mandates for monetary policies, HANK research is bound to attract attention from constituencies that today want to assign goals to Central Banks that involve redistribution and reallocation.

[15]Most of them are diplomats so they'd say 'improve' not 'replace' .

Some of these goals are so foreign to what Keynes (1924, 1936) advocated that perhaps we should remove the K from HANK.

The descriptive modeling branch of the HANK research project brings new interest to tools, old and new. An old tool whose promise was long neglected or unrealized was invented by Koopman (1931). He constructed an operator that, by measuring appropriate functions of the state (some eigenfunctions), maps a lower-order non-linear dynamic system into a higher-order linear system. In doing so, the Koopman operator makes the optimal linear control theory that has long been a mainstay of rational expectations econometrics (see the introduction to Lucas and Sargent (1981)) applicable to an interesting class of non-linear models. It also brings links to functional autoregressions, in particular to some recent applications of machine learning to fluid dynamics in the form of dynamic mode decompositions, called DMD. DMD can be a fast way of estimating a first-order functional VAR by applying a singular value decomposition (SVD) to a tall-skinny data matrix X. See Tu et al. (2014) and Brunton and Kutz (2022).

Appendix

A Keynes as historian and prognosticator

I describe some of the monetary-fiscal policy controversies that Keynes had in mind when, in the passage cited in section 2, he said that "There is no record of a prolonged war or a great social upheaval which has not been accompanied by a change in the legal tender, but an almost unbroken chronicle in every country which has a history, back to the earliest dawn of economic record, of a progressive deterioration in the real value of the successive legal tenders which have represented money."[16] While section A.1 indicates that Keynes's "unbroken chronicle" characterization doesn't describe 19th century US outcomes well, it does capture how contending interests sought to turn Federal monetary policy decisions to their advantage. Section A.2 then documents how 20th century US outcomes confirmed Keynes's pessimism about "progressive deterioration in the real value of the successive legal tenders which have represented money".

[16] See the chapters on historical evidence in Keynes (1930, 1931b).

A.1 19th century US episodes

I confine this subsection to controversies that raged during the US Civil War (1861-1865) and the 15 years that followed the end of the War. Monetary-fiscal policies that contributed to outcomes during those years were influenced by statesmen's memories and understandings of earlier wars that had unleashed similar forces. Thus, rehearsals for those Civil War monetary-fiscal controversies occurred during and following the US War for Independence from 1776 to 1783 and again during and following the US War of 1812.[17] After glancing at some of the 19th century outcomes, I'll turn briefly to some US data from the 20th century. All of these episodes illustrate how the issues and forces described by Keynes had preoccupied US fiscal-monetary policy makers and their constituencies. I'll reproduce graphs of U.S. price levels and *ex post* returns on Federal public debt data assembled by George Hall of Brandeis University.[18]

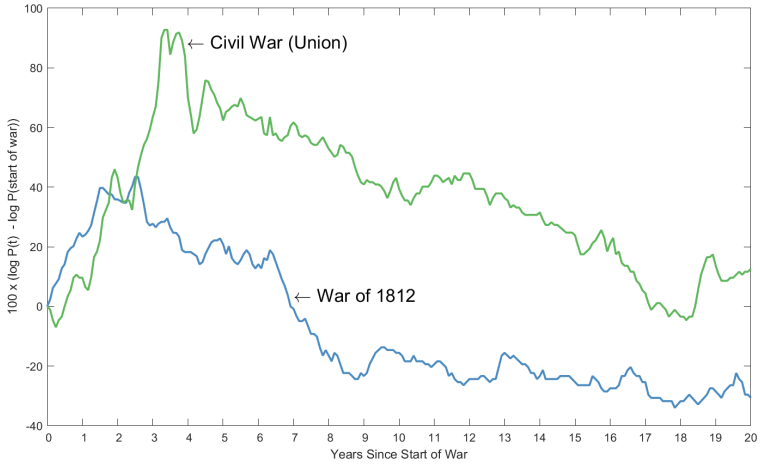

Figure 1: Log Price Level

Figure 1 shows the logarithm of the US price levels during and after two big 19th century US wars, the War of 1812 and the Civil War. Figure 2 shows cumulative returns on a representative portfolio of US federal debt during and after those two wars. I'll focus on the Civil War. In 1862, the Union (northern) government left the gold standard and issued an inconvertible paper currency called greenbacks that it made a legal tender for most, but not all, debts, public and private. By 1864, the greenback had depreciated to about 40

[17]The War of 1812 outcome pattern reversed one that characterized the US War of Independence and its aftermath, a consequence of deliberate policy choices described by Hall and Sargent (2014) and Sargent (2012).

[18]For many more details see Hall and Sargent (2021) and Hall and Sargent (2014).

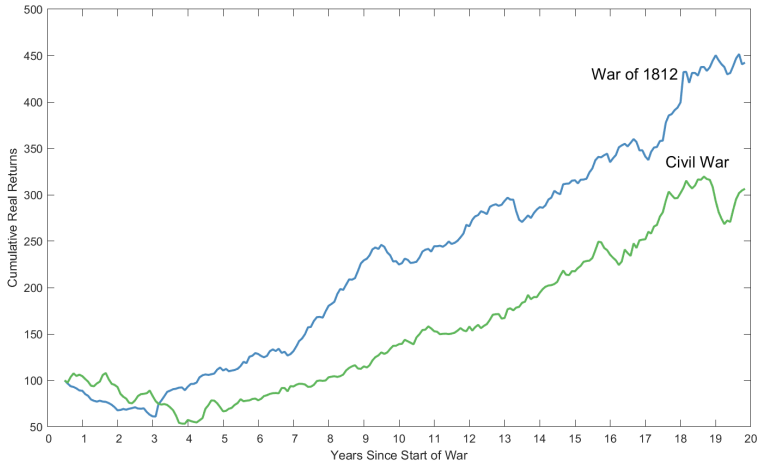

Figure 2: Cumulative Real Returns

gold cents per greenback dollar, the gold-greenback exchange rate moving with outcomes of battles between Union and Confederate forces. The war ended in April 1865 with gold at 60 cents per greenback dollar. The price level was denominated in greenbacks; its movements mirrored those of the gold-greenback exchange rate. Our graphs show how the price level rose during the war and how federal creditors received low returns during the war, but high returns afterward. This pattern echoed the US experience during the War of 1812.[19]

From 1865 until 1879 and beyond, controversy swirled about whether to make the greenback convertible into gold, and at what exchange rate. It was especially heated from 1865 until March 1869 when Ulysses S. Grant was inaugurated as President.[20] Congress had left ambiguous whether it intended the face value of important classes of bonds (the famous 5-20's) to be paid in greenbacks or gold. Many private bonds had been dominated in greenbacks, including many railroad bonds. Advocates for creditors contended with advocates for debtors, provoking debates cutting across both major political parties and regions. The following words from two of the highest authorities are examples of the contending positions. As an advocate of "rescheduling" (i.e., partial default) we cite

> There seems to be a general concurrence as to the propriety and justness of a

[19]It also echoed experience in England during and after the Wars with France from 1797 to 1815. It differed from US experience during and after the US War of independence in ways that persuaded policy makers during the War of 1812 to do things differently. See Hall and Sargent (2014).

[20] Newcomb (1865) criticized Union monetary policy for provoking adverse redistributions consequent on its making inconvertible greenbacks a legal tender. His book is remarkable in a number of ways, one being how far he gets deploying the labor theory of value, another being an information-theoretic analysis of optimal taxation in which ingredients of Ramsey and Mirlees theories are both present.

> reduction in the present rate of interest ... The lessons of the past admonish the lender that it is not well to be over-anxious in exacting from the borrower rigid compliance with the letter of the bond.
>
> President Andrew Johnson (Fourth Annual Message December 9, 1868)

Against President Johnson and most of the Democratic Party, the Republican party advocated honoring all public debts:

> We denounce all forms of repudiation as a national crime; and national honor requires the payment of the public indebtedness in the utmost good faith to all creditors at home and abroad, not only according to the letter, but the spirit of the laws under which it was contracted.
>
> Republican Party Platform, plank 3, 1868.

Republican candidate General Ulysses S. Grant won the 1868 election. At his inauguration, he said:

> A great debt has been contracted in securing to us and our posterity the Union. The payment of this, principal and interest, as well as the return to a specie basis as soon as it can be accomplished without material detriment to the debtor class or to the country at large, must be provided for. To protect the national honor, every dollar of Government indebtedness should be paid in gold, unless otherwise expressly stipulated in the contract. Let it be understood that no repudiator of one farthing of our public debt will be trusted in public place, and it will go far toward strengthening a credit which ought to be the best in the world, and will ultimately enable us to replace the debt with bonds bearing less interest than we now pay.
>
> U.S. Grant, first inaugural address, March 4, 1869.

The Republicans delivered on Grant's promise in a process full of improvisations and postponements that unfolded during and after the two Grant administrations (1869-1877). The US Treasury made greenbacks convertible at par into gold starting on Jan 1 1879.[21]

[21] It remained there until 1933. Proposals to redistribute via inflation resurfaced often after 1879.

A.2 Twentieth century US outcomes

The preceding graphs and quotes provide examples of some of the same disputes about manipulating the price level to redistribute wealth among creditors and debtors that concerned Keynes (1924, ch. 1). In those US nineteenth century episodes, a coalition that did not want to use the price level to redistribute wealth from nominal creditors to nominal debtors had prevailed. Those 19th century episodes are exceptions to Keynes's characterization of secular debasement of legal tenders as an "unbroken chronicle in every country which has a history". Economic historians have presented many more such exceptions in the 19th and earlier centuries. But outcomes in the 20th century differed from the 19th century. Figures 3 and 4, respectively, show the log of price level and cumulative real returns on the US Federal debt from the beginnings of World Wars I and II.

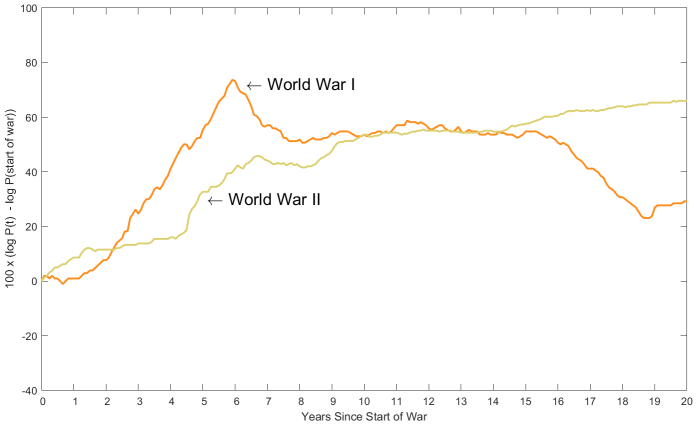

Figure 3: Log Price Level

Price levels rose persistently after the starts of both world wars. The Great Depression from 1929 until the end of our graph rise after WWI temporarily reversed the rise. That reversal, and the redistributions to nominal creditors from nominal debtors that accompanied it, had concerned Keynes (1924, ch. 1) as well as Fisher (1933). Those concerns inspired monetary-fiscal policies of President Franklin Delano Roosevelt, policies explicitly designed to redistribute from nominal creditors to nominal debtors.[22]

[22]See Edwards (2018).

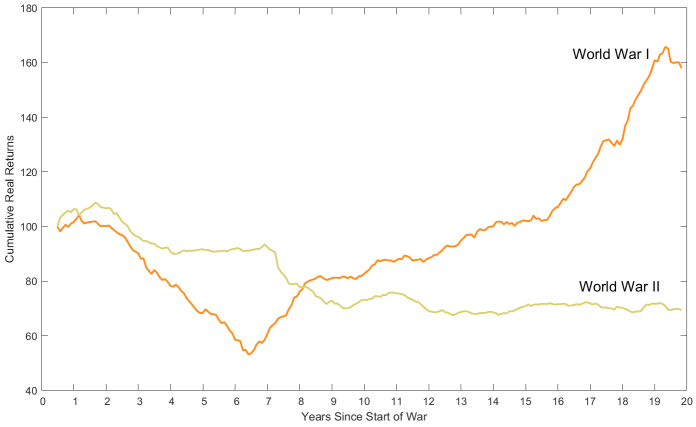

Figure 4: Cumulative Real Returns

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